MECHANICS FOR ENGINEERS

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Statics and Kinetics

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PREFACE

In contributing this book to the flood of available literature on the subject, the authors claim nothing new except an arrangement and a selection of material, a combination of problems with text for the exercise of the best pedagogy, and an emphasis upon subjects important to engineering students with corresponding omission or reduction of the less important ones. All of this is in conformity with modern methods of teaching, and is intended to meet the progress made by Universities in the development of their Engineering curricula according to the demands of industrial requirements.

Applied Mechanics is not an easy science to teach, nor is it simple to learn and apply. This is partly due to the fact that, since it is a fundamental subject, it must be taught no later than the sophomore year, when the student is still scholastically immature. It should therefore be given in small doses, with sufficient informative problems to stimulate intellectual digestion.

The average student cannot possibly assimilate, and work the problems necessary to understand all of the principles in a comprehensive treatment of Statics and Kinetics in a course of 60 to 80 class-room hours. Hence the effort to make a brief book.

The material is arranged so that the easiest comes first, with only enough definitions presented to cover that part. Notions of velocity and acceleration in pure rectilinear and rotative motion, and the relations between them are given just before the topics of Kinetics. It is thought that a logical and orderly arrangement has been made in a path easiest for the student to follow.

Subjects such as *the friction circle, the motion of projectiles*, etc., included in many modern works on Applied Mechanics, probably only because it is conventional to do so, are omitted here because the average engineering student, after graduation, has no use for them, and because the time gained by omitting their study may be applied more usefully to other concepts and practice.

Problems are interspersed through the text in such a way that the student cannot read very far before he meets an illustrative problem

(that is, one whose solution is given) followed by several others for him to solve. In this way he gets that practice in applying demonstrated principles which is absolutely essential to a working knowledge of the subject. It is as futile to study Mechanics without working problems, as to get a book idea of how to swim without entering water.

Many of the problems bring out principles through their solution that otherwise would need to be formulated in the text. This stimulates the reasoning powers of the student, and fixes the principles involved more firmly upon his memory and understanding.

The physical interpretation of all the equations and laws presented in this book is emphasized. The problems are designed so as to stress this important feature.

A system of notation is presented such that nowhere under the subjects of Statics and Kinetics is the same symbol used with different meanings. At the same time, familiar symbols commonly used in other works are adhered to, and the notation otherwise suggests its own meaning.

Attention of the instructor is called to Part XV, dealing with the analysis of the units in equations of mechanics. It is recommended that students refer to this from time to time and follow the ideas therein suggested.

Answers to all problems have been worked by slide rule and therefore the third significant figure is uncertain. The student should be reminded of the fact that this is generally accurate enough for Engineering purposes. It is for this reason, throughout the text π is written 3.14 and the acceleration of gravity 32.2.

The authors of this work have found that it is not possible to teach all of it well in 64 class-room hours applied thus: 52 hours for recitation and discussion of problems, 4 hours for monthly written quizes, 8 hours for review at end of term, $1\frac{1}{2}$ hours allowed for outside preparation per hour of class-room instruction; maximum number of students per instructor, 20. Under these conditions, these classes have been able to cover all of the text and problems except under the heads of the force and funicular polygons, roof trusses, impulse, momentum and impact. There is, therefore, presumably enough material in the book for any instructor to make a choice of subjects in the time mentioned or, in a greater time, to cover it all. For a brief course, the articles numbered in italicized type may be omitted.

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February, 1928.

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TABLE OF NOTATION

		Fundamental Units
m, n, o, p, q, s, t, u	Points on a diagram or sketch	
0, 1, 2, 3, 4, 5, etc.	Points on a force or funicular polygon	
c, c', c''	Center of gravity	
x, y, z	Coordinates of a point	L
P, Q, S	Areas, volumes, or solids on a sketch	L^2, L^3
k	A ratio	None
	FORCES	
F, F', F'', etc.	Applied forces	F
F_x, F_y	Rectangular components of a force	F
F_e	Equilibrant force	F
F_r	Resultant force	F
$\overline{F}$	Friction force	F
F_c	Centripetal force	F
F_c'	Centrifugal force	F
F_i	Inertia force	F
F_u	Force per unit area (pressure)	FL^{-2}
R, R', R'', etc.	Reactions	F
R_n	Normal reaction	F
R_k	Kinetic reaction	F
W, W'	Weights of bodies	F
L, l	Length	L
r, r'	Radius	L
r_g	Radius of gyration	L
T	Time	T
M	Mass	$FL^{-1}T^2$

		Fundamental Units
$\overline{M}=F\times L$	Moment of a force	FL
$\overline{M}=F\times r$	Torque	FL
f, f'	Static and kinetic coefficients of friction	None
ϕ, ϕ'	Static and kinetic angles of friction (radians)	None
β, θ, γ	Plane angles (radians)	None
V, V', V''	Various linear velocities	LT^{-1}
V_1, V_2	Initial and final velocities	LT^{-1}
A	Acceleration	LT^{-2}
ω	Angular velocity	T^{-1}
ω_1, ω_2	Initial and final angular velocities	T^{-1}
α	Angular acceleration	T^{-2}
N or r.p.m.	Number of turns per minute	T^{-1}
N'	Number of turns	None
N''	Number of turns per sec.	T^{-1}
I_x, I_y	Rectangular centroidal moment of inertia	L^4
I_z	Polar centroidal moment of inertia	ML^2 or L^4
I_x', I_y', I_z'	Moments of inertia about axes other than centroidal	ML^2 or L^4
$\overline{W}=L\times F$	Work	LF
K	Linear kinetic energy	LF
K'	Angular kinetic energy	LF
Hp.	Horse-power	FLT^{-1}
Ihp.	Indicated horse-power	LFT^{-1}
Bhp.	Brake horse-power	LFT^{-1}
E	Efficiency	None

For values of Fundamental Quantities F, L, T, see Art. 10 and Part XV.

MECHANICS FOR ENGINEERS

PART I

DEFINITIONS AND GENERAL PRINCIPLES

1. **The science of mechanics** covers, broadly, kinematics and dynamics. Kinematics has to do only with motion between bodies without regard to the size of the bodies or the magnitude of the forces causing motion. Dynamics is the science of forces acting on bodies and their effect to produce equilibrium or motion. It embraces statics and kinetics; the one having to do with balanced forces and hence a state of rest or uniform motion, and the other, unbalanced forces with the resulting motion of the bodies to which they are applied.

Thus dynamics includes the science of the elastic strength of materials, and the kinetics of fluid bodies, the former being specifically referred to as Mechanics of Materials, and the latter, Hydraulics.

This book will treat only with dynamics of rigid bodies; that is, the statics and kinetics of (assumed) non-elastic and non-fluid bodies.

2. **Force** is an action between two bodies, either causing or tending to cause change in their relative rest or motion. The commonest English unit of measurement is the pound.

The most familiar example of a force is, perhaps, muscular effort, and this, indeed, has been, for many generations and in many industries, the sole motivation of crude machines. Other actions, recognizable as forces, and falling within the scope of the preceding definition, are, for example: gravitation, friction, elastic resistance of materials, magnetism, etc. It will be observed that most of the forces dealt with in this book act by direct contact of one body with another, but this is not true of gravity, centrifugal force, or the so-called "inertia force" which will be discussed later. The definition of force requires concepts of matter (of which bodies are composed) and of motion and rest.

3. Matter, or material, is that which fills space. A working, though restricted, definition is: anything to which (when present in sufficient quantity) the senses of sight or feeling, or both, respond. It is recognized by such properties as weight, elasticity, inertia, etc. A limited and definite portion of matter is said to be a body.

4. Motion between two bodies is the relation, in space, that exists when a straight line joining any point on the one body to any point on the other varies in length.

5. Rest is a similar relation, when all such lines remain constant in length.

6. Space is measured in terms of length. The units of length used in this book are: feet, inches and miles. A unit of area is measured in units of length squared, and of volume, units of length cubed.

7. Weight, in this book, is the force of gravity between the earth and any body at or near the surface of the earth. A more exact definition is given in Art. 76, p. 71. This force varies slightly with the altitude and the latitude of the location of the body. The weight of a body at any given locality is directly proportional to the amount of material of which it is composed.

8. Mass is that property of matter by virtue of which it resists change in its state of rest or motion. This property is also called inertia. Since the resistance to such change is proportional to the amount of matter involved, mass is often defined, and conceived of, as "quantity of matter." Its units are discussed under Art. 77.

9. Time is the relation between past, present and future events. Its unit in the fundamental mathematical relations of mechanics is the "second" and will be so used in this book. In engineering formulas derived from such relations, the "minute" and the "hour" are more used than the "second."

10. The fundamental quantities in mechanics, as here presented are:

Force,	F,	in pounds
Length,	L,	in feet
Time,	T,	in seconds

from which may be derived

Mass,	M,	for units see Art. 77

In this work the unit force is standardized by the force of gravity upon a concrete body or its prototype. The system of measurement is

thus called the "gravitational system." In other works, notably those employing the metric system, the unit force is derived from the standard mass, as that force which will produce a unit change of velocity if applied to a unit mass. This is called the absolute system, and, in practice, is only employed when the metric system is in use. Mass, then, is a fundamental unit, and force a derived unit.

All compound quantities may be reduced to these fundamental units. Unless otherwise stated, F, L, and T are in the units named.

PROBLEM 1. State mathematically the fundamental units of the relation that one cubic foot of water weighs 62.3 lbs. *Ans.* F/L^3.

PROBLEM 2. State an example of statics. Of kinetics.

PROBLEM 3. Name as many as occur to you of different kinds of forces falling under the definition of Art. 2.

PROBLEM 4. Is an invisible gas "matter" as defined under Art. 3?

PROBLEM 5. A body, P, weighs 10 lbs. at a given locality and has m units of mass. Another body, Q, weighs 20 lbs. at the same locality. How many units of mass has Q?

PROBLEM 6. Which has the greater mass, a cubic foot of wood or a cubic foot of lead?

PROBLEM 7. A certain volume of wood weighs the same, at the same locality, as a certain smaller volume of lead. How do their masses compare? Would their masses be different at a higher altitude? Would their weights?

PROBLEM 8. Write the fundamental units of "gallons," "miles per hour," "cubic feet per pound," pressure in "pounds per square foot."

11. Concentrated forces and point of application. When a concentrated force is transmitted from one body to another, there must be a definite, though possibly very small, area of contact subjected to the transmitted force. For example, consider the conditions imposed upon a metal sphere, acting as a ball bearing. A machine member of this sort, if it could be perfectly rigid and bearing on equally rigid surfaces, would be in contact with those surfaces at two mathematical points situated at the extremities of a diameter of the sphere, no matter how much force was applied to squeeze the ball between the surfaces. There is, however, no such thing as a truly rigid body; that is, a body which does not yield, or become deformed when stressed. In the case of the ball bearing just cited the material of the ball is under a compression force which causes a yielding of the material, thus changing the mathematical point of contact to a minute circular area over which the force is distributed.

In many problems in mechanics similar in physical conditions to the preceding illustration, this area is assumed to be a point, which is called the point of application of the force.

This applies not only to mathematical *points* of contact, but also *lines*

of contact, such as would be obtained by supporting a perfectly rigid body on one edge of a triangular prism, equally rigid. Although the force between the two bodies in this instance is distributed over a mathematical line, the solution of problems involving similar physical conditions is just the same when it is assumed that the force is concentrated at a point. This point is the intersection of the line of contact with a plane perpendicular to the line, when all the forces involved may be considered to act in the selected perpendicular plane.

Concentrated forces, then, are those which may be considered to have a single point of application.

12. Distributed forces are those to which large areas are subjected. As an example, the pressure of steam upon the piston of a steam engine is a force distributed over the whole area of the piston.

In the mechanics of rigid bodies, a distributed force may be replaced by a single concentrated force having the same effect as the distributed force.

13. Center of gravity. Gravity acts on all the mass particles of a body alike as far as engineering structures are concerned. It is, therefore, a distributed force. Its action on *rigid* bodies is such that it can always be taken as a concentrated force passing through a point called the center of gravity of the body. With symmetrical homogeneous objects, this point is the center of symmetry. (See Art. 35.)

14. The line of action of a force is that line along which the force tends to cause motion. The following principles should be understood:

The effect of a force acting upon a *rigid* body is the same no matter at which point on the line of action the force is applied.

In many problems the line of action and the direction of a force may be foretold from its nature or from the physical conditions. For example:

Gravity always acts vertically downwards, and its line of action may be taken through the center of gravity.

The force tending to keep taut a cable or flexible connector must have its line of action coincident with the center line of the cable or connector; if the cable or connector, for the condition of the problem, may be considered as being without weight. For, if a body is at rest under the action of two concentrated forces, these forces must be opposed and have a common line of action.

Friction always acts tangentially to the surfaces of contact. Its line of action is parallel to the motion, or tendency to motion, of the body upon which it acts. Its direction is always opposed to such motion.

15. Equilibrium exists when the various forces on a body are balanced. In general, the body is then at rest; but equilibrium may also prevail when the balancing forces produce uniform motion.

16. The resultant of a group of unbalanced forces is a single force producing the same result or tendency in magnitude and direction as the group. In statics it is often convenient to replace a group of two or more forces acting on a body by their resultant, which, with other applied forces, maintain equilibrium. If there are no such other applied forces, motion ensues, and the problem is one of kinetics.

17. The equilibrant of a group of forces is the single force having the same magnitude and the same line of action as the resultant, but the reverse direction, and is the force necessary to maintain equilibrium with an unbalanced set of forces.

18. Reaction. When a force acts from a body, P, to another body, Q, the body Q resists with an equal and opposite force. Thus if one compresses a spring by bearing down upon it with the hand, the muscular force exerted is resisted by the tendency of the spring to resume its original configuration, that is, by its elasticity.

In general, one or more forces may be considered the *active* forces, and one or more the *reactive* forces or reactions. For example, a ladder carrying the weight of a man, and bearing against a floor and a wall, transmits a force to the floor and another to the wall. These supports must respond with equal and opposite forces (reactions) upon the ladder.

In statics an applied force must have an equal and opposite reaction.

19. Graphical representation of forces. Vectors. Forces acting upon a structure or machine part are represented by lines drawn on a sketch or diagram of the structure or part. These lines are supplemented by arrow heads to indicate the direction of the forces they represent. If the lines are drawn to scale so that their lengths are in proportion to the magnitudes of the denoted forces, they are called "vectors." A vector, then, is a *line* on a drawing or diagram representing a magnitude and a direction.

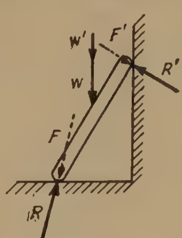


Fig. 1.

20. Forces "on" and "from" a body. In Art. 18 the case of a ladder bearing the weight of a man and supported at wall and floor was cited. The forces *on* the ladder are: Gravity on the ladder itself, the weight of the man, and the reactions *from* the floor and wall. The "space diagram" for this is shown by Fig. 1, the vectors marked W, W', R and R' representing the forces in the order just given. The force *on* the

floor and *from* the ladder is opposite to, and in the same line as the reaction R; and is represented in Fig. 1 by the dotted vector, F. F is the active force given by the bottom of the ladder to the floor. Similarly with the force, F', on the wall.

When considering the effect of various forces on a body, the student must be careful to take into account only the forces *on* it, and not to confuse them with the forces *from* it.

21. The space diagram is a drawing which may or may not be to scale, representing a structure or machine, or a part of either, the forces on which are represented by vectors. Following the definition of "vectors," the space diagram thus shows lines to indicate the direction and magnitude of the forces involved, either to scale, or dimensioned in units of force and angle of inclination from some reference line (as the horizontal or vertical).

When the space diagram is made for a *part* of a structure or machine, that part only is drawn, and *only* the forces affecting that special part are indicated. These vectors represent the weight of the part, any applied forces to the part, and the reactions at any point of contact between the part considered and any other portion of the structure or machine, or with any external support or obstruction.

Thus, there may be drawn space diagrams for a mechanical contrivance as a whole, or for any of its parts. When the space diagram is made for the whole, it may be considered a rigid body, and the vectors represent gravity and externally applied forces including reactions wherever the contrivance is in contact with any external support or obstruction.

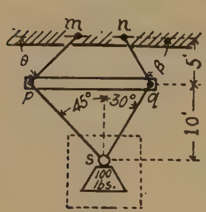


FIG. 2.

PROBLEM 9. Consider a body weighing 100 pounds suspended by the system of cords of Fig. 2, so arranged that the spreader, pq (weight of which is to be neglected) is horizontal. Draw the space diagram for the joint s.

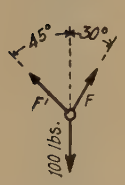


FIG. 3.

Solution. Figure 3 shows the required space diagram, the section included in dotted lines being the part considered. The 100-lb. vector is produced by gravity which causes a vertical pull downward. The cords sq and sp are in tension, and the forces they exert at s are along their center lines. Therefore, upward to the right and making an angle of 30° with the vertical is drawn the vector F, which represents the tension in the cord sq. Upward to the left and making an angle of 45° with the vertical is drawn the vector F', which represents the tension in the cord sp.

PROBLEM 10. In Fig. 2, if the spreader (of negligible weight) is horizontal and the angles $\theta = 45°$ and $\beta = 60°$ draw the space diagrams for the following:

(*a*) The joint *p*,
(*b*) The joint *q*,
(*c*) The cord *mp*,
(*d*) The horizontal member *pq*.

PROBLEM 11. A man is seated in the middle of a hammock. Draw the space diagram of the hammock.

PROBLEM 12. A rigid metal sphere weighing 25 lbs. rests on a rigid surface. A vertical downward force of 10 lbs. is applied at its uppermost point. Make the space diagram for the sphere. Scale: 1 inch = 10 lbs.

PROBLEM 13. A man is hanging from the center of a trapeze bar by one hand. Draw the space diagram for the trapeze; for the man.

PROBLEM 14. A man is hanging from the bar of a trapeze by both hands so that each arm is carrying half his weight. Draw the space diagram for the trapeze; for the man.

PROBLEM 15. A man lifts a heavy object with a crow bar (lever) suitably fulcrumed. Draw the space diagram for the crow bar.

PROBLEM 16. Which are the distributed and which the concentrated forces in Problem 12?

PROBLEM 17. Same as Problem 12 except that the 10-lb. force is upward.

PROBLEM 18. A horizontal push of 10 lbs. is given the sphere along a horizontal diameter, *mn*, applied at the extremity *m*. Will the effect be the same as with a horizontal pull at the extremity *n*?

PROBLEM 19. Does equilibrium exist in Problems 12, 17 and 18?

22. The solution of problems in statics requires a careful inspection of the forces in action. As far as may be, a predetermination should be made of the following:

(*a*) The points of application of all forces upon the body considered.
(*b*) The lines of action and the directions of the forces.
(*c*) The magnitudes of the forces.

Given the physical conditions and the magnitude and direction of one or more of the forces, it is required to find the magnitude and direction of one or more unknown forces holding the body in equilibrium.

As stated in Art. 14 (which re-read) some of these data may be deduced from the physical conditions involved in the problem. In this connection the following theorem should be thoroughly understood:

23. Theorem. *If the surfaces of contact of two bodies in equilibrium are held together under an active force and its reaction, and there is no friction in effect, then the line of action of the two forces must be perpendicular to the surfaces.*

PROBLEM 20. A cylinder rests on two surfaces as shown in Fig. 4. Draw the space diagram assigning directions to all forces acting *on* the cylinder.

Solution. Since there is no tendency of the cylinder to rotate on the surfaces, there is no friction in effect and the reactions must be perpendicular to each plane and radial to the cylinder as shown by Fig. 5, R and R'. The only other force *on* the cylinder is gravity indicated by W, passing through the center of symmetry.

PROBLEM 21. Fig. 6 represents a davit for supporting a boat the weight of which is indicated by W. The davit is restrained from any but horizontal circular motion by the socket, m, and the sleeve, n, shown in section. What are the directions of the reactions at the sleeve and socket?

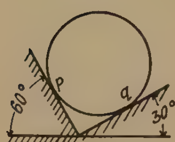


FIG. 4.

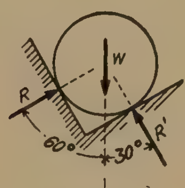


FIG. 5.

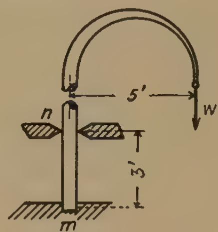


FIG. 6.

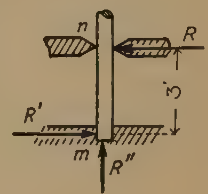


FIG. 7.

Solution. Imagine the support at n to be insufficiently strong and that the structure gives way under the force, W, revolving vertically around the socket, m, to the right. The reaction at n, restraining this motion, must, therefore, be to the left. Since there obviously can be no vertical motion of the davit relative to the sleeve, no friction is in effect, and the reaction at n must be perpendicular to the surface of contact at the sleeve as shown by R, Fig. 7.

A similar reasoning applies to the socket, m, from which it is concluded that there is a reaction, R', directed to the right and perpendicular to the left-hand wall of the socket. But if these reactions were the only ones, the davit would have a vertical downward motion. Consequently there must be a third reaction, R'', directed upward.

In many problems it is very helpful to imagine part of a structure giving way and then to consider how a force must be directed at that part to maintain equilibrium.

PROBLEM 22. In Fig. 1, if there were no friction at the wall or floor, could equilibrium exist? Why?

PROBLEM 23. Assume that there is a cleat nailed to the floor as shown by Fig. 8, so that the ladder cannot slip to the left. Sketch the space diagram for the ladder assigning directions to all forces.

PROBLEM 24. Three rigid metal cylinders, each 5 ins. in diameter and weighing 25 lbs., are supported in a channel piece 10 ins. wide as shown in Fig. 9. Draw the space diagrams for each ball and the channel iron.

PROBLEM 25. Draw the space diagram for a gate weighing 50 lbs. and supported as shown by Fig. 10.

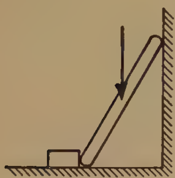


FIG. 8.

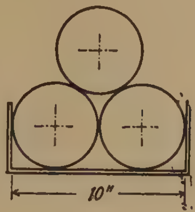


FIG. 9.

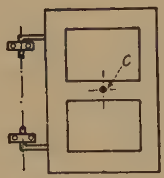


FIG. 10.

PROBLEM 26. A body of considerable weight is pressed against a horizontal surface by a force, F, inclined 45 degrees from the vertical. If there is inconsiderable friction, can equilibrium exist? Why?

24. The moment of a force is a measure of its tendency to produce rotation of the body upon which it acts about some axis. The axis is taken as perpendicular to a plane containing the circular path of any point of the body, assuming rotation actually to take place. In a space diagram representing coplanar forces, this axis is therefore represented by a point which is called the *center of moments.*

The tendency of a force to rotate a body is greater the greater the distance of the line of action of the force from the center of moments as is illustrated in the use of the simple lever. Numerically, then, the moment of a force, F, is the product of its magnitude and the perpendicular distance, L, from the center of moments to the line of action of the force. Its units in this book are "pound-feet" unless otherwise specified; but "pound-inches," "ton-feet," etc., are also used. The symbol $\overline{M}$ will be used to signify "moment," and $\overline{M} = FL$.

The perpendicular distance, L, is often referred to as the moment arm.

The tendency to rotate may be either right hand or left hand. This may be taken care of algebraically by using plus signs for the one rotation and minus signs for the other.

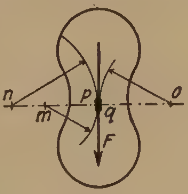


FIG. 11.

The center of moments may be taken anywhere in space regardless of the physical possibility of rotating the body considered about the selected center. Referring to Fig. 11, the irregular outline is supposed to be the cross-section of a body formed by a plane which passes through the line of action of the applied force, F. Select any point, m, in this plane, and from

it draw a perpendicular to the line of action of F. At the intersection, the two consecutive points, p, q, may be considered to lie on the arc of a circle centered at m. This is equally true of points n and o, considered as centers. If the force, F, produces motion of the body in the direction of its line of action then, *for the instant*, the motion may be considered one of rotation about any of the points m, n or o.

If the center of moments is taken on the line of action of a force, the moment of that force is zero, since $L = 0$.

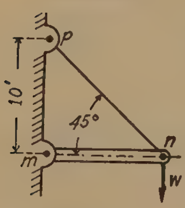


Fig. 12.

Problem 27. If in the problem of the boat davit, Fig. 6, the weight, W, is 600 lbs., and the reaction, R, is 1000 lbs., what are the moments of all the forces about the point m as center?

Solution. Of W, $600 \times 5 = 3000$ lb.-ft. plus. Of R, $1000 \times 3 = 3000$ lb.-ft. minus. Of R', $R' \times 0 = 0$. Similarly the moment of $R'' = 0$.

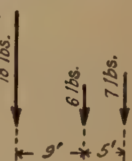


Fig. 13.

Problem 28. In Fig. 12 the tension in the cable is 1000 lbs. What is the moment of this force about m? *Ans.* 7070 lb.-ft.

Problem 29. In Fig. 13 what are the moments of the forces shown about a point located 3 ft. to the left of the 18-lb. force? *Ans.* 54, 72, 119 lb.-ft.

PART II

PARALLEL COPLANAR FORCES

25. Theorems. Resultant of coplanar parallel forces. Using plus for forces acting in one direction, and minus for those acting in the opposite direction; and a similar notation for their moments about any point in their plane as indicated in Art. 24, then, (excepting "couples" to be considered later, Art. 28).

(*a*) *The resultant of a set of such forces has a magnitude equal to that of the algebraic sum of the forces of the set, and its line of action is parallel to those of the set, and in the same plane.*

(*b*) *The resultant has a moment about any point in the plane of the forces equal to the algebraic sum of the moments of the forces of the set about the same point.*

PROBLEM 30. Find the resultant, F_r, in magnitude, location and direction of the two parallel forces, F and F', acting upon a body, P, as shown by Fig. 14.

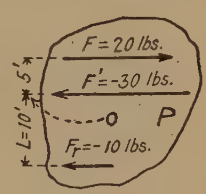


FIG. 14.

Solution. Call forces acting to the right, plus and forces acting to the left, minus and moments tending toward clockwise rotation, plus and anti-clockwise, minus.

Since the algebraic sum of F and F' is 20 lbs. − 30 lbs. = − 10 lbs. the resultant must act toward the left.

To apply the principle of moments, select a center of moments, o, on the line of action of one of the forces, say F', in order to eliminate the moment of that force. Then, calling the moment arm, L, of the resultant about this center, and $\overline{M}_r$ the moment of the resultant

$$F_r \times L = \overline{M}_r = (-10) \times L = 20 \times 5 = 100,$$
$$L = 100 \div (-10) = -10 \text{ ft.}$$

The minus sign indicates that the moment arm, L, of F_r is *below* the center of moments, o, and the resultant, F_r, is located as shown by its vector, Fig. 14.

An inspection of Fig. 14 shows that if the body were to revolve about an axis through the point o, under forces, F and F', the moment causing

rotation would be due to F only, and would be clockwise and equal to 100 lb.-ft. The resultant, F_r, would produce exactly the same motion about o since its moment is also clockwise about o, and equal to

$$(-10 \text{ lbs.}) \times (-10 \text{ ft.}) = 100 \text{ lb.-ft.}$$

Problems. For problems 31, 32, 33, 34, and 35 given below, find the magnitude, location and direction of the resultants of the given parallel forces.

Problem 31. Two 10-lb. forces acting downward and 6 ft. apart.
Ans. 20 lbs. downward and midway between the forces.

Problem 32. F = 16 lbs. and F' = 24 lbs., both downward and 8 ft. apart.
Ans. 40 lbs. downward, 4.8 ft. from F and 3.2 ft. from F'.

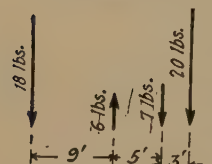


Fig. 15.

Problem 33. F = 16 lbs. downward and F' = 24 lbs. upward and 8 ft. apart.
Ans. 8 lbs. upward, 24 ft. from F and 16 ft. from F'.

Problem 34. Forces as shown in Fig. 13.
Ans. 31 lbs. downward, 4.9 ft. to right of 18-lb. force.

Problem 35. Forces as shown in Fig. 15.
Ans. 39 lbs. downward, 9.84 ft. to right of 18-lb. force.

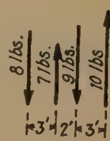


Fig. 16.

Problem 36. Are the forces shown in Fig. 16 in equilibrium? Why?

26. Theorems. Coplanar parallel forces in equilibrium:

(*a*) *The algebraic sum of the forces equals zero.*

(*b*) *The algebraic sum of the moments of the forces about any point in their plane equals zero.*

These theorems follow from Art. 25 (*a*) and (*b*), since, if the resultant of a set of parallel forces is reversed, equilibrium results and the algebraic sum of the forces and of their moments each equals zero.

When solving problems in equilibrium by these theorems a useful method is to write on one side of the equation all the forces acting in one direction with plus signs, and on the opposite side of the equation all the forces acting in the opposite direction and again with plus signs. Similarly, an equation involving moments may be written. According to this principle the previously stated theorems may be expressed:

(*c*) *If a set of coplanar parallel forces is in equilibrium, the sum of the forces acting in one direction equals the sum of the forces acting in the opposite direction, and*

(*d*) *The sum of the moments of the forces tending to produce clockwise rotation about any point in their plane equals the sum of the moments of the forces tending to produce anti-clockwise rotation about the same point.*

PROBLEM 37. Find the equilibrant of F and F' in Fig. 14.

Solution. From Problem 30, F_r is 10 lbs. to the left and 10 ft. below F'. Therefore, by Art. 26, F_e, the equilibrant, must be located in the line of action of F_r and in the opposite direction, that is, to the right, or, by (*c*)

$$F_e + F = F', \quad \text{or} \quad F_e + 20 = 30,$$

whence
$$F_e = 10 \text{ lbs.},$$

by (*d*) taking moments about a point on F':

$$F \times 5 = F_e \times L, \quad \text{or} \quad 20 \times 5 = 10 \times L,$$

whence
$$L = 100 \div 10 = 10 \text{ ft.},$$

which checks with the results of Problem 30, Art. 25.

27. **A simple beam** is one supported freely at the ends and is generally horizontal. The applied forces are usually vertical; consequently the reactions at the supports are also vertical. The magnitudes of these reactions may be found by the principles of Art. 26. It is convenient to choose for the center of moments, an axis at one support, in order to eliminate from the moment equation, the reaction there. The equation of moments then yields the reaction at the other support. The remaining reaction may be obtained by taking an axis at the other support as a center of moments. It is well to check the solutions by the principle of summation of forces, Art. 26 (*c*).

In the following problems the weight of the beam is neglected and the loads are concentrated.

PROBLEM 38. Fig. 17 represents a simple beam loaded as shown. The magnitudes of the reactions, R and R', are to be found.

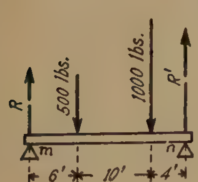


FIG. 17.

Solution. Since the forces are in equilibrium:

$$R + R' = 1000 + 500 \text{ [Art. 26, (}c\text{)]}.$$

Taking moments about an axis at the left support:

$$500 \times 6 + 1000 \times 16 = R' \times 20 \text{ from which } R' = 950 \text{ lbs.}$$

Taking moments about an axis at the right support:

$$1000 \times 4 + 500 \times 14 = R \times 20 \text{ from which } R = 550 \text{ lbs.}$$

To check: 950 lbs. + 550 lbs. = 1500 lbs.

PROBLEM 39. Interchange the locations of the 500 and 1000-lb. forces of the simple beam of Fig. 17. What are the magnitudes of the two reactions?

Ans. 700 lbs., 800 lbs.

PROBLEM 40. Fig. 18. Find R and R'. *Ans.* 4333 lbs., 2667 lbs.

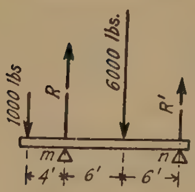


FIG. 18.

PROBLEM 41. Two men are carrying a 300-lb. weight hanging from a slender rod so that their handholds are 12 ft. apart. It is desired that one man carry twice the weight carried by the other man. Where should the weight be placed along the rod? *Ans.* 4 ft. from the man carrying the heavier load.

PROBLEM 42. Fig. 19. Derive a relation between R, W, L and L', to show the variation of R as L' increases from zero to L.

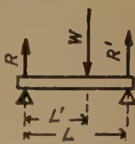


FIG. 19.

A freely supported beam may be loaded so that the reaction is downward instead of upward. The equation of moments for this reaction will then yield a negative value, indicating that the solved for reaction is actually opposite in direction to that assumed.

PROBLEM 43. Fig. 18. Find R and R' if the point of support, m, is moved 5 ft. to the right.

Solution. Taking the new position of m as the center of moments the new moment arms of R' and the 6000-lb. and 1000-lb. loads are 7 ft., 1 ft. and 9 ft. respectively. Hence,

$$R' \times 7 + 1000 \times 9 = 6000 \times 1,$$
$$R' = -3000 \div 7 = -429 \text{ lbs.},$$

and R' therefore points downward. Another way of arriving at this conclusion is to observe that the moment of the 1000-lb. force about the new position of m (namely, 9000 lb.-ft.) is in excess of the opposite moment of the 6000-lb. force (namely, 6000 lb.-ft.). Consequently anti-clockwise rotation would result unless the support at n were placed on the top side of the beam, causing a downward reaction.

PROBLEM 44. Assume a beam supported at points m and n and loaded as shown in Fig. 20. Find the directions and magnitudes of the reactions, R and R'.

Ans. 3000 lbs. up; 2000 lbs. down.

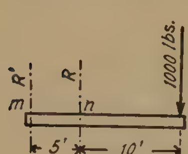


FIG. 20.

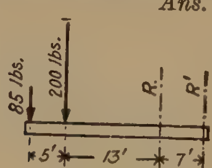


FIG. 21.

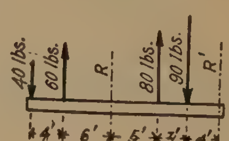


FIG. 22.

PROBLEM 45. Fig. 21. The lines of action of the reactions for the beam shown are indicated by the dotted lines. Find the magnitudes and directions of the reactions.

Ans. 590 lbs., 875 lbs.

PROBLEM 46. Fig. 22. Same questions as Problem 45.

Ans. 23.33 lbs. up; 33.33 lbs. down.

If the beam is of uniform weight, per unit of length, this additional load upon the supports may be accounted for by assuming the weight of the beam to act through its center of gravity.

PROBLEM 47. Fig. 23. Find R and R' if the weight of the beam is 50 lbs. per foot of length.

Solution. The total weight of the beam is $50 \times 12 = 600$ lbs. which is shown by the vector with a line of action midway between the ends of the beam. Taking as center of moments, the left-hand point of support, the weight of the beam, as shown by the figure, has a moment arm of 4 ft. Hence,

$$R' \times 7 = 300 \times 6 + 600 \times 4 + 400 \times 1,$$

and $$R' = 655 \text{ lbs.}$$

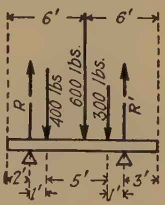


FIG. 23.

The same method applies when the beam has a uniformly distributed load over all or part of its length.

PROBLEM 48. If the beam of Fig. 18 is loaded uniformly with a weight of 58 lbs. per foot, as well as with the concentrated loads shown, what are the magnitudes of the reactions R and R'?

Ans. 2976 lbs., 4952 lbs.

PROBLEM 49. If the beam of Fig. 20 is loaded with a uniform load of 150 lbs. per foot, as well as with the concentrated load of 1000 lbs. as shown, what are the magnitudes and directions of the reactions R and R'?

Ans. 6375 lbs. up; 3125 lbs. down.

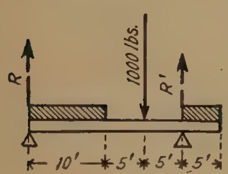


FIG. 24.

PROBLEM 50. The beam of Fig. 24 carries a uniform load of 80 lbs. per linear foot at the shaded portions. Neglecting the weight of the beam, what are the magnitudes of the reactions R and R'?

Ans. 800 lbs., 1400 lbs.

28. Couples. Two parallel forces equal in magnitude and opposite in direction constitute a *couple*. (See Fig. 25). The perpendicular distance, L, between the forces is called the *arm* of the couple. The product of one of the forces and the arm is called the *moment of the couple*, and is equal to the algebraic sum of the moments of the forces of the couple referred to any point in the plane.

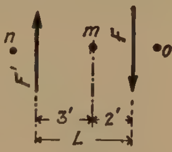


FIG. 25.

PROBLEM 51. In Fig. 25, if $F = F' = 10$ lbs. and $L = 5$ ft., the moment of the couple $= 10 \times 5 = 50$ lb.-ft. Compare this with the algebraic sum of the moments of F and F' about a point, m, located 2 ft. from F and 3 ft. from F' (between the forces of the couple). Then $10 \times 2 + 10 \times 3 = 50$ lb.-ft. as before.

PROBLEM 52. Using the couple of Fig. 25 with the values given in Problem 51 find the sum of the moments of the forces of the couple about a point, n, located 7 ft. to the left of F'; about a point, o, 2 ft. to the right of F.

PROBLEM 53. Prove, in general, that the moment of a couple is constant regardless of the location of the center of moments.

29. **Theorem.** *The effect of a couple on a body is rotation or a tendency to rotation, and is the same regardless of the location of the couple in its plane.* This follows from Problem 53.

An important corollary to this theorem follows:

If a body is in uniform rotation, then the resultant of all the forces acting upon it is a couple or zero.

This may not be immediately obvious. For instance, it may be imagined that a wheel would rotate on a fixed axis if a single tangential force is applied. Further consideration shows that an equal and opposite force is set up, by the application of the tangential force, at the bearing holding the axis. Thus, consider a wheel with a tangential force applied, as shown by Fig. 26, in which uniform motion is assumed. The wheel is supported by a rod running through the center of its hub. There is a reaction, R, opposite and equal to F, for if there were not, the wheel would have a motion of translation downward, instead of rotation. Similar reasoning is true no matter how the force, F, is inclined.

FIG. 26.

Gravity force upon the wheel does not change the preceding conclusion, since it must always be balanced by an equal and opposite reaction from the supporting rod.

PROBLEM 54. Fig. 27 represents a plan view of a capstan. The 50-lb. forces are delivered by two men operating the capstan. Assume one man to move a foot nearer the center of the capstan and the other man one foot further away, what is the sum of the moments of the forces about o? Compare this answer with the moment of the couple as shown in the figure.

PROBLEM 55. The men operating the capstan of Fig. 27 decrease their effort to 40 lbs. each. If the turning moment is to remain constant at what distance from the center should their efforts be applied? *Ans.* 3.75 ft.

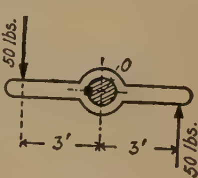


FIG. 27.

30. **Theorem.** *The magnitude of the forces and the moment arm of a couple may be changed at will provided that the moment of the couple is kept constant and in the same direction.* Thus the effect of a clockwise couple composed of two 10-pound forces acting 4 feet apart is exactly the same as another clockwise couple composed of two 8-pound forces acting 5 feet apart or of a clockwise couple composed of two 80-pound forces acting 0.5 foot apart.

PROBLEM 56. Convert the couple exerted by the men on the capstan of Fig. 27 into one having the same effect but having an arm of 3 ft. Into one having the same effect but having forces of 25 lbs. each.

PROBLEM 57. What is the moment of the forces shown in Fig. 28 about the point o? Explain the seeming paradox.

PROBLEM 58. Prove the theorem at the beginning of this article.

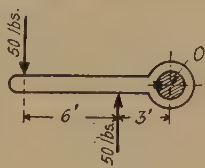


FIG. 28.

31. Theorem. *The moments of several coplanar couples may be added algebraically and the resultant moment obtained.* Any point in the plane of the couples may be taken as the common center of moments for all of the couples. Each couple will have a moment about any point, o, equal to the product of one of the forces of that couple and the arm of the couple. Hence the sum of the moments of the couples is the resultant moment of all the couples about the point, o, as a center.

PROBLEM 59. What is the resultant moment of the couples shown in Fig. 29?

Solution. By adding the moments of the couples algebraically we get the moment of the resultant couple: $40 \times 4 + 80 \times 5 - 50 \times 6 = 260$ lb.-ft., which is a clockwise couple.

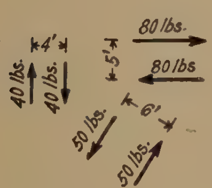


FIG. 29.

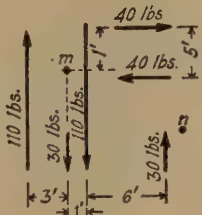


FIG. 30.

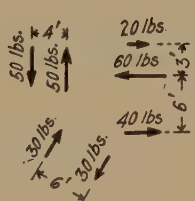


FIG. 31.

PROBLEM 60. Find the resultant of the forces shown in Fig. 30 about m; about n. *Ans.* 430 lb-ft. clockwise couple.

PROBLEM 61. Find the resultant of the forces shown in Fig. 31. *Ans.* 200 lb.-ft. anti-clockwise couple.

From the preceding theorem it follows that, if the sum of the moments of several coplanar couple equals zero, equilibrium exists.

PROBLEM 62. Refer to the davit shown in Figs. 6 and 7. If the weight, W, is 900 lbs., what two couples hold the structure in equilibrium? Show on a space diagram.

Problem 63, Fig. 8. Take the length of the ladder as 12 ft. and its inclination from the floor as 60 deg. The vector, W, indicating the weight of a man half way up the ladder plus the weight of the ladder is 190 lbs. Draw a space diagram showing the couples holding ladder in equilibrium, assuming no friction force. Give magnitudes of couple forces and arms.

32. Theorem. *The resultant of a couple and a single force, F, is a force, F', equal and parallel to F and acting at such a distance from F and in such a direction as to make the moment of F' about a point on F equal to the moment of the original couple.*

Problem 64. Find the resultant of the couple and force shown in Fig. 32.

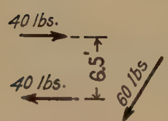


Fig. 32.

Solution. The moment of the couple is 40 lbs. × 6.5 ft. = 260 lb.-ft. clockwise. Assume this couple to consist of two forces of 60 lbs. each and an arm of $4\frac{1}{3}$ ft. (Art. 30.) Place this couple with relation to the 60-lb. single force as shown in Fig. 33. (See Art. 29.) Two of the 60-lb. forces balance leaving as a resultant the single 60-lb. force acting $4\frac{1}{3}$ ft. from the given single force and in the direction shown.

Fig. 33.

Problem 65. Find the resultant of a clockwise couple of two 15-lb. forces with a 4-ft. arm and a horizontal force of 20 lbs. acting to the right. *Ans.* A 20-lb. force to the right 3 ft. above the original 20-lb. force.

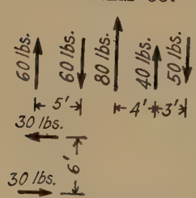


Fig. 34.

Problem 66. Find the resultant of an anti-clockwise couple of two 30-lb. forces and a 2-ft. arm, and a horizontal force of 20 lbs. acting to the right.

Problem 67. Find the resultant of the forces shown in Fig. 34.

Ans. 70-lb. force upward, 4.43 ft. to left of 80-lb. force.

Problem 68. Prove the theorem at the beginning of this article by generalizing Problem 64. Use the notation given in the theorem, and let $\overline{M}$ be the moment of the given couple.

PART III

CENTER OF GRAVITY

33. Center of parallel forces. Let F, F', F'', etc., be a set of parallel forces, not necessarily lying in one plane. Select a set of rectangular axes, ox, oy and oz, with which to coordinate the lines of action of these forces. As shown by Fig. 35 the oy axis is drawn parallel to the forces. F and F' may be replaced by a single force lying in their plane. Similarly, this single force may be combined with F'', and so on until a single resultant, F_r, remains. Any point on the line of action of F_r with coordinates $\bar{x}$ and $\bar{z}$ (see Fig. 35) is called the center of the given parallel forces.

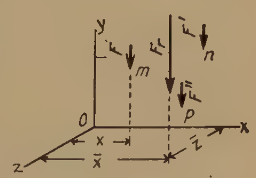


FIG. 35.

Mathematically, these coordinates may be obtained by taking moments, first about the axis, oz, and then about the axis, ox. Designating with corresponding primes the coordinates of the given forces, then, taking moments about oz:

$$F_r\bar{x} = Fx + F'x' + F''x'' + \ldots,$$

and since $F_r = F + F' + F'' + \ldots,$

$$\bar{x} = \frac{Fx + F'x' + F''x'' + \ldots}{F + F' + F'' + \ldots}.$$

A similar deduction yields a corresponding value for $\bar{z}$.

Note that the location of F_r, with relation to the given forces, is independent of the origin of coordinates, and also independent of the direction of the parallel forces, the direction being the same for all.

Let it now be assumed that these forces have definite points of application, m, n and p, Fig. 35.

Required, to find a point, c, at which F_r may be applied, no matter in what direction the given parallel forces, and hence F_r, are inclined; it

being understood that the points of application m, n, and p have fixed locations with respect to each other.

Fig. 36 shows the forces applied to points m, n, and p in a horizontal direction parallel to ox. The resultant, F'_r of the applied forces, in a horizontal direction, equals the resultant, F_r, in a vertical direction; since the magnitudes of the applied forces are the same in the two cases.

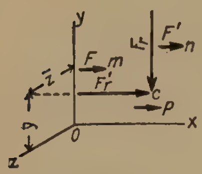


Fig. 36.

When the forces are vertical (Fig. 35), moments about ox give

$$\bar{z} = \frac{Fz + F'z' + F''z''}{F_r}.$$

When the forces are horizontal (Fig. 36) moments about oy give

$$\bar{z} = \frac{Fz + F'z' + F''z''}{F'_r}.$$

These two values of $\bar{z}$ are identical since $F_r = F'_r$ and the z dimensions of m, n, and p (that is, moment-arms) remain unchanged. Consequently F_r intersects F'_r in a point, c.

This point is called the center of the given forces.

To make the demonstration general, suppose the horizontal forces, F, F', F'' in Fig. 36 be *not* parallel to axis, ox. To prove that F_r intersects F'_r at point c, for this case, it is only necessary to select the coordinating axes in Fig. 35 so that ox will be parallel to the new lines of action of the forces shown in Fig. 36. The values of x and z for points m, n, and p with this change of axes are different but the location of these points and that of point c, with respect to each other, remain unchanged.

If the applied forces are not swung through 90 degrees, but some other angle, a similar proof requires that the coordinating plane, xy, be parallel to a plane determined by the first and second position of the applied forces. (See Fig. 37.) Axis ox must be perpendicular to a line, ss, parallel to the first position of the forces; and axis oy perpendicular to a line, tt, parallel to the second position, thus making inclined axes of oy and ox. Otherwise the demonstration is the same.

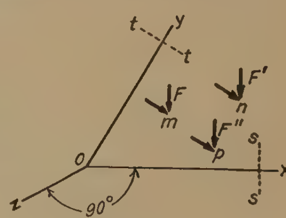


Fig. 37.

34. Center of gravity. Referring again to Figs. 35 and 36, it is seen that point c may be definitely located by its distances from the yz, the xz and the xy planes; that is, the dimensions $\bar{x}$, $\bar{y}$, and $\bar{z}$ are determinable.

With the forces vertical, taking moments about oz

$$\bar{x} = \frac{Fx + F'x' + F''x'' + \ldots}{F + F' + F'' + \ldots}.$$

With the forces horizontal, taking moments about oz,

$$\bar{y} = \frac{Fy + F'y' + F''y'' + \ldots.}{F + F' + F'' + \ldots}.$$

With the forces horizontal, taking moments about oy,

$$\bar{z} = \frac{Fz + F'z' + F''z'' + \ldots}{F + F' + F'' + \ldots}.$$

$\bar{x}$ and $\bar{y}$ could also be found by taking the forces perpendicular to plane, xy, and then taking moments about axis oy for $\bar{x}$, and about ox for $\bar{y}$. Also, $\bar{z}$ could be found by taking the forces as vertical and moments about ox (Fig. 35).

The point, c, may thus be definitely coordinated with reference to any chosen set of axes.

Assuming that m, n, p, etc. are points of application of forces, F, F', F'', etc., *on a rigid* body, the center of these forces, c, is independent of their inclination, so long as they remain parallel, as just proved. Conversely, if the forces remain with the same parallelism and the body to which they are applied be inclined to any position with respect to the forces, the center, c, remains unchanged with respect to the body, provided only that the points of application, m, n, p, etc., have fixed locations with respect to the body.

Let us assume that the points, m, n, p (Fig. 35) are material particles, having weights, W, W', W'' respectively. So long as the relative positions of these material particles are fixed, then the center of the gravity forces, W, W', W'' may be coordinated exactly as in Art. 33. This center is then called the center of gravity or centroid of the material particles. In general, the center of gravity of a body is that point through which passes the resultant of all the forces of gravity acting on the particles of the body, no matter in what position the body is held with relation to the earth. (In engineering problems forces of gravity are taken as parallel.)

It is necessary to determine the center of gravity of bodies to locate the weight vectors and other forces to be considered in later problems.

PROBLEM. 69. Given the following data applying to Fig. 35:

	Weight	x	y	z
m	1.8 lbs.	1″	2″	2″
n	1.2	4″	4.5″	6″
p	0.75	3″	5″	7″

Find $\bar{x}$, $\bar{y}$, and $\bar{z}$. *Ans.* $\bar{x} = 2.36$, $\bar{y} = 3.4$, $\bar{z} = 4.28$.

35. Density and center of gravity of solids. *The density* of a body is the mass per unit volume of the material of which the body consists. In engineering, units of mass are not commonly used; consequently density is spoken of in terms of the weight of a unit volume of the material. The weight varies with the locality (Art. 7), but the variation is so small (maximum one-half of one per cent in the United States) that, for most engineering purposes, density expressed in terms of weight per unit volume may be used with sufficient accuracy.

The units of density may then be taken in pounds per cubic foot, pounds per cubic inch, etc. When it is said that the "density of a material is W pounds per cubic foot," the full significance is that one cubic foot of the material has such a mass that its weight is W pounds at a stated locality.

PROBLEM 70. An engine flywheel made of cast iron has a volume of 12,380 cu. ins. The density of cast iron may be taken as 0.26 lb. per cu. in. How much does the flywheel weigh? What is the density of its material in pounds per cubic foot?

A homogeneous body is one any particle of which has the same density as any other.

Center of gravity of solids. Consider any irregular homogeneous body referred to rectangular coordinates. Let dQ represent a differential volume of this body. Then, if D is its density, the weight of the differential volume is $dW = DdQ$. The summation of the moments of all the gravity forces about the oz axis is $\int xDdQ$. Then,

$$W\bar{x} = \int xDdQ = D\int xdQ.$$

The resultant of all the gravity forces is

$$W = D\int dQ = DQ.$$

Dividing the first equation by the second gives

$$\bar{x} = \frac{D}{DQ}\int xdQ = \frac{1}{Q}\int xdQ.$$

The same deduction applies to $\bar{y}$ and $\bar{z}$. Consequently the center of gravity of a homogeneous body is the same, regardless of the material of which it is composed, since the density factor cancels in the equations for the coordinates. From the above demonstration it is obvious that the center of gravity must coincide with the center of symmetry of the body, if there is such a center; and it must lie on a line or plane of symmetry. This follows from the fact that $\int xdQ$, $\int ydQ$, $\int zdQ$ become zero, the plus and minus values balancing, when the origin of coordinates is on a center of symmetry.

PROBLEM 71. Locate the center of gravity with reference to the point, o, of the iron casting shown with dimensions in Fig. 38. Use the center line as the ox axis.

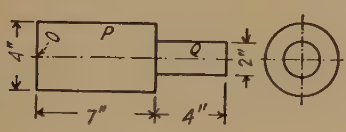


FIG. 38.

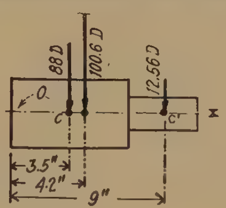


FIG. 39.

Solution. Since the body is symmetrical with respect to the line, ox (Fig. 39), $y = 0$ and $z = 0$. It remains to find $\bar{x}$. Call the large cylinder, P, and the smaller cylinder, Q. The center of gravity of P is on its axis at c, Fig. 39; of Q on its axis at c'. (Art. 35.)

Weight of $P = D \times 7 \times 3.14 \times 4 = 88D$ lbs.
Weight of $Q = D \times 4 \times 3.14 \times 1 = 12.56D$ lbs.

These weights are shown by vectors in Fig. 39.

Moment of weight of P about o is $88D \times 3.5'' = 308D$ lb.-in.
Moment of weight of Q about o is $12.56D \times 9'' = 113D$ lb.-in.

The sum of these moments $= 421D$ lb.-in.

Dividing this sum by the total weight of P and Q, which is $88D + 12.56D = 100.6D$, gives the distance of this resultant from o or

$$\bar{x} = \frac{421D}{100.6D} = 4.2 \text{ ins.}$$

PROBLEM 72. Locate the center of gravity of two cylinders dimensioned as in Problem 71, but eccentric as shown in Fig. 40, with respect to the point, *o*.

Ans. 4.2 ins., −0.125 in., 0.

PROBLEM 73. Two prisms with a common axis, *ox*, form a solid body as shown in Fig. 41. Locate its center of gravity with respect to the point, *o*.

FIG. 40.

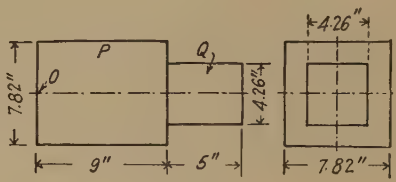


FIG. 41.

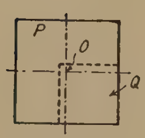


FIG. 42.

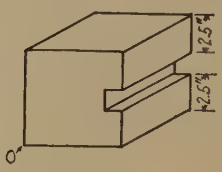


FIG. 43.

PROBLEM 74. The two prisms of Problem 73 are arranged as shown in Fig. 42 with their lower rear corners coinciding. Locate the center of gravity with respect to the point, *o*.

Ans. 5.50 ins., −0.25 in., −0.25 in.

PROBLEM 75. Locate the center of gravity of a 6-in. cube with a 1 × 1-in. slot, as shown in Fig. 43, with respect to the point, *o*.

Ans. 2.928 ins., 3 ins., −3 ins.

36. The center of gravity of an area or a line is a mathematical conception necessary to certain engineering problems. A notion of the physical meaning of center of gravity applied to such a figure may be obtained by imagining the area to be a very thin homogeneous plate or the line a very thin homogeneous wire. The force of gravity will then be proportional to the area of the plate or to the length of the wire.

In such cases the word *centroid* may be used instead of center of gravity. Centroid is a generic term used to denote center of gravity of volumes, areas and other geometric figures.

PROBLEM 76. Locate the centroid of the area shown in Fig. 44 with reference to the point, *o*.

Solution. Consider the *L*-shaped figure to consist of two rectangles, *P* and *Q*,

Fig. 45, with centroids located at c' and c'' respectively. If these areas (rectangles) were thin plates of unit weight per square inch of area, their weights would be:

$$\text{Of } P, \quad 4 \times 10 = 40,$$
$$\text{Of } Q, \quad 3 \times 5 = 15.$$

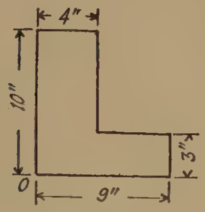


FIG. 44.

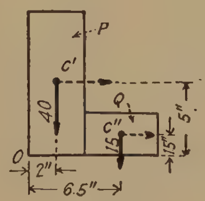


FIG. 45.

Shown as vectors in Fig. 45. Taking moments about o and dividing by the total weight gives:

$$\bar{x} = \frac{(40 \times 2) + (15 \times 6.5)}{40 + 15} = 3.23 \text{ ins.}$$

Assuming the gravity to act as shown by the dotted vectors, then, as before:

$$\bar{y} = \frac{(40 \times 5) + (15 \times 1.5)}{40 + 15} = 4.04 \text{ ins.}$$

In the following, locate the centroid with respect to the point, o, as shown in each figure.

PROBLEM 77. Divide the areas of Fig. 44 into two rectangles as shown in Fig. 46 and check the results of the Problem 76.

PROBLEM 78. Fig. 47 representing an area. *Ans.* 0, 3.27 ins.

PROBLEM 79. Fig. 48 representing a line. *Ans.* 2.8 ins., 5.25 ins.

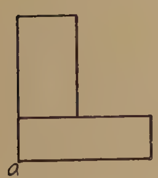


FIG. 46.

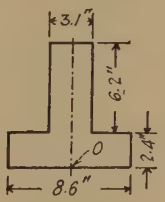


FIG. 47.

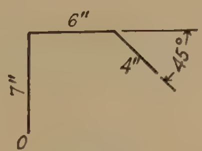


FIG. 48.

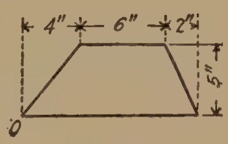


FIG. 49.

PROBLEM 80. Fig. 49 representing an area. (The centroid of a triangle is at the intersection of the medians, or a distance from any side taken as a base, equal to one-third the altitude from that base.) *Ans.* 6.44 ins., 2.22 ins.

Problem 81. The shaded area shown by Fig. 50. *Ans.* 4.97 ins., 4.85 ins.
Problem 82. The area shown in Fig. 51. *Ans.* 1.65 ins., 2.62 ins.

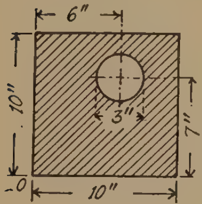


Fig. 50.

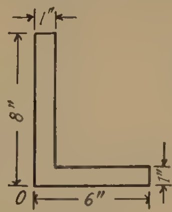


Fig. 51.

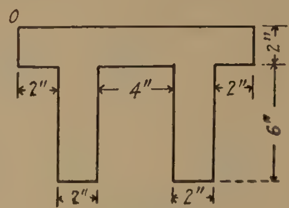


Fig. 52.

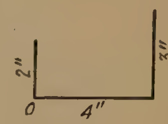


Fig. 53.

Problem 83. Fig. 52, an area. *Ans.* 6 ins., −3 ins.
Problem 84. Fig. 53, a line. *Ans.* 2.22 ins., 0.722 in.

37. Theorems. *The volume of a solid of revolution equals the product of the generating area and the circumference described by the centroid of the area.* A similar theorem applies to a surface of revolution, it being understood that in both cases the generating area or line is not intersected by the axis of rotation and lies in the same plane as this axis.

Problem 85. If the trapezoid of Fig. 49 is revolved about an axis 10 ins. below and parallel to its base, what is the volume of revolution generated? *Ans.* 3450 cu. ins.

Problem 86. What is the volume of revolution generated by the shaded area of Fig. 50 when revolved about an axis 1 in. below its base and parallel to it? *Ans.* 3410 cu. ins.

Problem 87. If the area of Fig. 52 is revolved about an axis parallel to its 12-in. side and 2 ins. above it, what is the volume of revolution. *Ans.* 1507 cu. ins.

Problem 88. If the line of Fig. 48 is revolved about a vertical axis 10 ins. to the left of the 7-in. side, what is the area of the surface of revolution? *Ans.* 1370 sq. ins.

PART IV

NON-PARALLEL COPLANAR FORCES

38. Resultant of non-parallel coplanar forces. It has been found by experiment that when two such forces act upon a body they may be replaced by a single force producing the same effect upon the body in magnitude and direction. This single force passes through the intersection of the lines of action of the two original forces, is coplanar with them, and can be found graphically as follows:

Referring to Fig. 54 the irregular outline represents a body acted upon

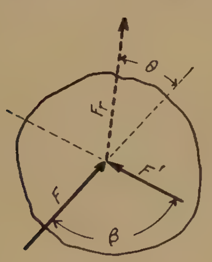


FIG. 54.

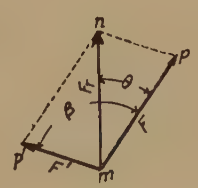


FIG. 55.

by two forces, F and F', at an angle, β, with each other. Figure 55 shows F and F' drawn to scale and inclined from each other by the angle, β. Complete the parallelogram, as shown by the dotted lines; then the diagonal, mn, represents the resultant, F_r, in magnitude and direction. This resultant may now be shown on the space diagram as indicated by the dotted vector in Fig. 54.

This principle may now be extended to apply to three or more unbalanced coplanar non-parallel forces, since the resultant of any two may be combined with a third, and so on until there is but a single force replacing the given forces.

It should be noted here that the equilibrant of two intersecting forces

may be found in the same manner, except that its direction is the reverse of that of the resultant.

Solve the following problems by the parallelogram of forces:

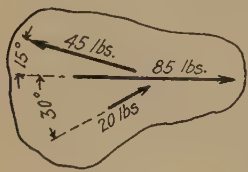


Fig. 56.

Problem 89. In Fig. 54 if F is 20 lbs. and F' is 40 lbs., and $\beta = 60°$, what is F_r in magnitude and direction? *Ans.* 53 lbs., $\theta = 41°$.

Problem 90. Find the resultant in magnitude and direction, of the forces shown in Fig. 56. *Ans.* 63 lbs., 19° from horizontal.

Problem 91. If, in Fig. 54, $F = 30$ lbs., $F' = 45$ lbs. and β is 45°, what is the equilibrant in magnitude and direction? *Ans.* 69 lbs., $\theta = 27°$.

39. The triangle of forces. Instead of constructing the parallelogram as shown in Fig. 55, the resultant may be obtained from the triangle mnp, the side pn being equal in magnitude and parallel in direction to the vector, F'. The following construction, shown by Fig. 57, is to be used.

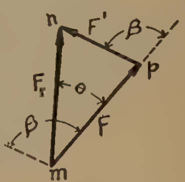


Fig. 57.

Lay off mp and pn parallel to F and F', respectively, in a force diagram, so that the vector directions do not oppose. The closing side of the triangle mn is the desired resultant.

Note: The vector representing the *resultant* or closing side of the figure, is *against* the direction of the other vectors, the word "direction" in this sense being taken as generally clockwise or anti-clockwise of a point travelling along the vectors as laid out in the figure. The figure is also called the **force diagram.**

Conversely, the vector representing the *equilibrant* is consecutively *with* the direction of the other vectors in the figure.

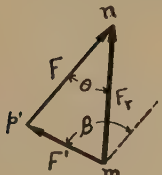


Fig. 58.

Following this construction, the other triangle of Fig. 55 might have been found as shown in Fig. 58, the resultant being the same in both arrangements.

A corollary to the principle of the triangle of forces is: If three forces are in equilibrium, their vectors, when represented in order, form a closed triangle.

Problem 92. Sketch the solution, not to scale, of Problem 89 by the triangle of forces, giving angles and marking vectors, with a clear notation, on both space and force diagrams.

Problem 93. Repeat Problem 92, but with the data of Problem 91.

40. Algebraic solution of the triangle of forces. Instead of scaling the vectors of the force triangle the resultant may be obtained from the trigonometric relation:

The square of one side of a triangle equals the sum of the squares of the other two sides minus twice the product of those sides times the cosine of the angle included between them.

PROBLEM 94. Check the answer of Problem 89 by the method of this article.
PROBLEM 95. Same as Problem 94 but for Problem 91.

41. Components of a force. The process described in Art. 38 may be reversed. That is, for a given resultant force, two or more forces called components, may be substituted to produce the same effect as that of the single given force. For example, in Fig. 54 if F_r be given, for it may be substituted F and F' found by constructing on the vector, *mn*, Fig. 55, the parallelogram with sides F and F'. For analytical purposes the angles β and θ may be chosen at random. Thus in Fig. 59, the angle β is 90 degrees, in which case the components, F and F', have different values from those in Fig. 55.

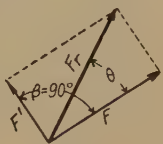


FIG. 59.

The process of determining for a given force, a pair of forces equivalent in their effect to the given force, is called "resolving into components" and each of the forces so found is called a "component force," or more briefly a "component."

When the angle included between the components is not a right angle, the components are called "oblique"; when the angle is a right angle they are called "rectangular components."

It is not necessary to construct the parallelogram to resolve into components. The triangle of forces suffices as shown in Fig. 60 drawn corresponding to Fig. 59. If the angle θ is so chosen with respect to a given pair of rectangular axes, *ox* and *oy*, that each component is parallel to one axis, they are called the x and y components, and are written F_x and F_y. In engineering work, the y axis is often vertical, in which case the components are referred to as the "vertical" and "horizontal" components.

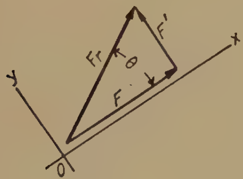


FIG. 60.

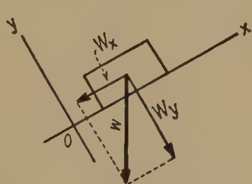


FIG. 61.

In engineering mechanics the directions of the components are frequently determined by the physical conditions. For example, Fig. 61 represents a block sliding down a frictionless plane under the action of gravity, W. Since motion occurs along the plane, there must be a component of W acting along the plane, as shown by W_x. Also, there must be a reaction between the block and the plane perpendicular to the plane. This forms the other component of W or W_y. In this case the ox axis is taken along the plane, and the oy axis perpendicular to the plane.

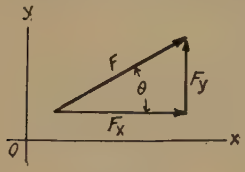


Fig. 62.

Figure 62 shows a force, F, resolved into rectangular components with reference to the axes shown. Then,

$$F_x = F\cos\theta \quad \text{or} \quad F = F_x \div \cos\theta,$$

and

$$F_y = F\sin\theta \quad \text{or} \quad F = F_y \div \sin\theta,$$

and

$$F^2 = F_x^2 + F_y^2 \quad \text{or} \quad F = \sqrt{F_x^2 + F_y^2},$$

and

$$\tan\theta = F_y \div F_x.$$

Problem 96. In Fig. 62 if the angle θ is 60° and F is 1000 lbs., what are the horizontal and vertical components? *Ans.* 500, 866 lbs.

Problem 97. A force of 500 lbs. is inclined from the horizontal at an angle whose tangent = 0.75. What are its vertical and horizontal components? *Ans.* 300 lbs., 400 lbs.

Problem 98. Fig. 63 shows a flat section of a boiler shell which is submitted to a distributed pressure of 12,000 lbs., which may be assumed to be a concentrated force as shown. What is the tension in the stay rod indicated in the drawing? Make a space diagram showing the two components of this tension and how they act. *Ans.* 13,860 lbs.

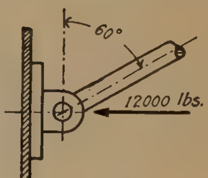


Fig. 63.

Problem 99. If the block of Fig. 61 is subjected to a force of 500 lbs. inclined upward to the right and at an angle of 15° with the plane, and the angle made by the plane with the horizontal is 30°, what are the x and y components of the force, referring to the coordinates shown? What are the horizontal and vertical components?

Ans. 483 lbs., 129 lbs.; 353 lbs., 353 lbs.

42. Pin-connected, two-force and three-force members. By "pin-connected," when applied to a structure or mechanism, is meant that the various members of the structure or mechanism may be considered as held together with small diameter loose pins inserted in holes in the mem-

bers. Under these conditions the members adjust themselves to the applied forces, justifying the assumption of point application and coplanar location of all forces acting on the members.

Figure 64 represents a boom, mn, pin-connected at the left end to a wall, and loaded near the right end with a weight of 1000 pounds. A cable, mp, maintains equilibrium, and there are definite reactions from the wall at points p and n. Now, if, instead of being pin-connected, the left end of the boom is imbedded in the wall, it may be made strong enough and sufficiently well gripped by the wall to support the load without the cable. The reaction at n is then entirely different. Suppose, next, that the cable, mp, be attached loosely, and then tightened with a turn-buckle. The tension in the cable, and therefore the reaction at p, can be made to vary from zero to any value until the structure fails, with accompanying varying values of the reaction at the wall. In such a case, then, it is necessary to assume pin-connections for a definite determination of reactions. This assumption is justified because in the real structure the members tend to yield until the reactions are the same as though pin-connected.

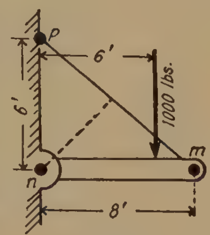


FIG. 64.

It simplifies the solution of problems involving equilibrium of non-parallel coplanar forces by considering *two-force* members as differentiated from *three-force* members.

Two-force members are members having forces acting at two points of application only. Therefore, for equilibrium, *the forces on a two-force member always act along the line of the member.*

Three-force members are members subjected to forces at three points of application, and having three different lines of action. Therefore, *the forces on a three-force member do not act along the line of the member.*

In actual practice a member is seldom a *two-force* member since it has weight, but it is often sufficiently accurate to assume that the weight of any individual member is negligible. Unless otherwise stated, all problems in this section will deal with structures the weight of whose members may be neglected.

43. Four-force (non-parallel) members generally can be analyzed by considering the forces as two pairs, and finding the resultant of each pair. These resultants must be equal, opposite, and colinear for equilibrium as indicated by Fig. 65 in which F, F', F'', and F''' are applied forces. If one of these forces, as F, is known in magnitude and direction, and the

directions of the remaining three also are known, it is possible to determine the magnitude of the remaining three forces as shown by the two parallelo-

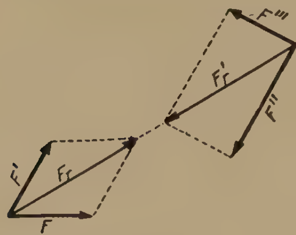


FIG. 65.

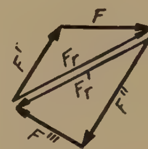


FIG. 66.

grams of forces, in which the diagonal, F_r, of the one equals F_r', the diagonal, of the other.

Solution by two triangles of forces is indicated by Fig. 66.

This principle can be extended to any number of forces, and, if desired, the force polygon applied (see Art. 51).

44. Conditions of equilibrium. If a set of coplanar forces are in equilibrium, and each force be resolved into rectangular components referred to the same set of axes in their plane, then:

(*a*) The algebraic sum of the x components equals zero, and the algebraic sum of the y components equals zero.

(*b*) The algebraic sum of the moments of the forces about any point in their plane equals zero.

It is convenient, in solving problems, to form the equations resulting from (*a*) and (*b*) according to the revised phraseology:

(*c*) The sum of the x components acting towards the right equals the sum of the x components acting towards the left.

(*d*) The sum of the y components acting upwards equals the sum of the y components acting downwards.

(*e*) The sum of the moments of the forces tending to produce clockwise rotation about the chosen center of moments equals the sum of those tending to produce anti-clockwise rotation.

The following theorem is of great assistance in the solution of three-force problems:

Theorem. *If a body is in equilibrium under the action of three forces, the lines of action of these forces must meet at a point.*

Proof: The resultant of any two of the three forces must be colinear,

opposite and equal to the third force or else equilibrium would not exist.

PROBLEM 100. Draw the space diagram and the force triangle for the boom shown in Fig. 64.

Solution. See Fig. 67. The boom is a three-force member and, by the theorem of Art. 44 (which re-read), these forces must intersect at a point. F has a line of action on the center line of the cable, and intersects the 1000-lb. vector, W, at the

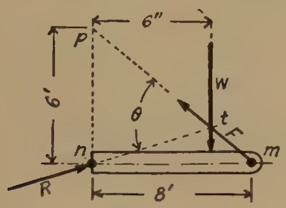


FIG. 67.

FIG. 68.

point, t. The line of action of R passes through its point of application, n, and also point, t, as shown. Fig. 68 shows the force triangle, the vectors arranged consecutively since there is equilibrium.

In Problems 101, 102, and 103 draw the space and force diagrams for the structures referred to.

PROBLEM 101. For the pin-connected boom shown in Fig. 69.

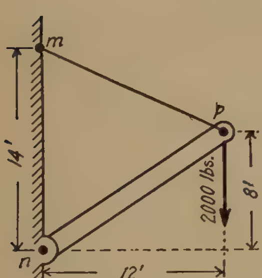


FIG. 69.

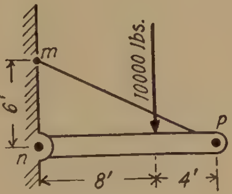


FIG. 70.

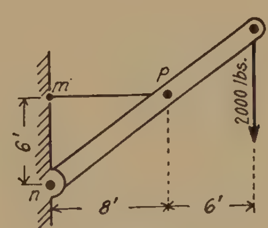


FIG. 71.

PROBLEM 102. For the wall at m and n in Fig. 70.
PROBLEM 103. For the pin-connected boom of Fig. 71.

45. Solution of problems. Knowing the physical conditions involved in a practical case of statics the force or space diagrams, or both, should

be drawn as in the preceding problems. Solution may be accomplished by any or all of the preceding principles. The first method is the graphical method (Art. 39) which is best employed as a check upon an algebraic solution (Art. 40). A modification of this is the method of similar triangles (Art. 46). We have, then, the force triangle, graphic, algebraic or geometric; the method of components (Art. 44); the method of moments (Art. 44). It will be found in some cases that only one of these methods may be applied for the easiest solution, or any two or three may be applied at different parts of the problem.

46. Solution by similar triangles. The force triangle is first drawn so that it is similar to a triangle on the space diagram whose dimensions are known or can be calculated. By proportionality of the vectors composing the force diagram to the corresponding distances on the space diagram the forces are obtained.

Problem 104. Find the forces *on* the boom shown in Fig. 64 by the method of similar triangles.

Solution. The space and force diagrams for this figure were drawn in the solution of Prob. 100 and are shown by Figs. 67 and 68 respectively. The force triangle of Fig. 68 is similar to the triangle ntp, Fig. 67, whose dimensions may be calculated. Thus

$$pn = 6 \text{ ft}; \; pt = 10 \times \tfrac{6}{8} = 7.5 \text{ ft.}; \qquad nt = 6.2 \text{ ft.},$$

$$F : 1000 = 7.5 : 6, \qquad \text{whence } F = 1250 \text{ lbs.}$$

$$R : 1000 = 6.2 : 6, \qquad \text{whence } R = 1033 \text{ lbs.}$$

Solve the following problems by the method of similar triangles.

Problem 105. In the pin-connected boom shown in Fig. 69 find the forces on the boom. Draw a space diagram showing the reactions at m and n in amount and direction.
Ans. 1910 lbs., 2058 lbs.

Problem 106. Find the reactions at m and n of the boom loaded as shown in Fig. 70.
Ans. 13,700 lbs., 14,900 lbs.

Problem 107. If the boat davit shown in Fig. 6 supports a weight of 900 lbs., what are the reactions at m and n?
Ans. 1500 lbs., 1750 lbs.

Problem 108. Find the forces on the boom shown in Fig. 71.
Ans. 4667 lbs., 5090 lbs.

47. Solution by components. Figure 72 represents, by the irregular outline, a body acted upon by the four coplanar forces, F, F', F'' and F'''

If these forces are resolved into rectangular components referred to axes, xx and yy, as shown in Fig. 73, then, from Art. 44,

$$F_y + F_y' = F_y'' + F_y'''$$

and

$$F_x + F_x''' = F_x'' + F_x'.$$

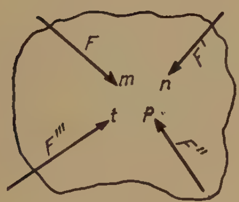


FIG. 72.

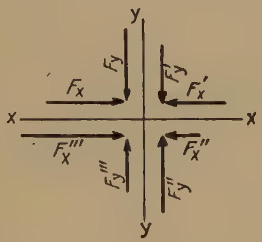


FIG. 73.

If one of the forces is unknown, both in magnitude and direction, its components, and hence its magnitude, may be found by this method. Its direction may then be determined from the relation (Art. 41),

$$\tan \theta = F_y \div F_x.$$

If the direction of all the forces of a set are known, two unknown forces may be obtained from these two equations of equilibrium.

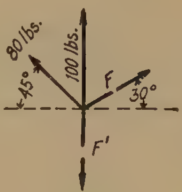


FIG. 74.

PROBLEM 109. In Fig. 74 the forces are in equilibrium. Find F and F'.

Solution. From the horizontal components,

$$F \cos 30 = 80 \cos 45, \text{ hence } F = 65.3 \text{ lbs.}$$

From the vertical components,

$$F' = 80 \sin 45 + 100 + 65.3 \sin 30 = 189 \text{ lbs.}$$

Solve the following problems by the method of components:

PROBLEM 110. In the boat davit of Fig. 6, take W as 900 lbs. and assume a single force at the point m. What are the reactions at m and n?
Ans. 1500 lbs., 1750 lbs.

PROBLEM 111. Find the reactions at n and p of the boom shown in Fig. 64.
Ans. 1033 lbs., 1250 lbs.

PROBLEM 112. The forces shown in Fig. 75 are in equilibrium. What are the magnitudes of F and F'? *Ans.* 134 lbs., 250 lbs.

Problem 113. Fig. 76 represents a pin-connected triangluar frame supported at points, m and n. Find the forces in the members, mp and pn, at the point, p, holding the 1000-lb. force in equilibrium.

Ans. 714 lbs., 810 lbs.

Problem 114. In Fig. 69 find the forces exerted by cable and boom at point, p.

Ans. 1910 lbs., 2058 lbs.

Problem 115. A weight of 100 lbs. is hung from two cables as shown in Fig. 77. What is the tension in the cables? *Ans.* 73.3 lbs., 89.8 lbs.

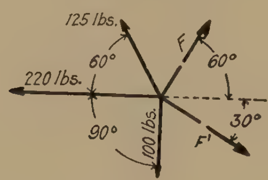


Fig. 75.

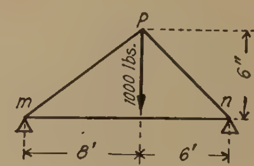


Fig. 76.

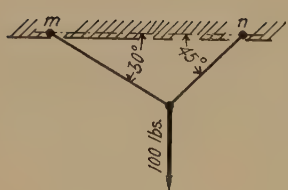


Fig. 77.

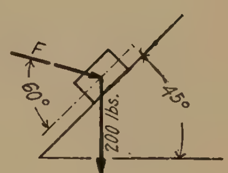


Fig. 78.

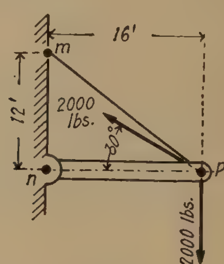


Fig. 79.

Problem 116. If there is no friction, what must be the force, F, inclined as shown in Fig. 78 to keep the block from sliding down? *Ans.* 283 lbs.

Problem 117. Fig. 79 shows a space diagram for a boom and supporting cable. A body weighing 2000 lbs. is supported by a cable running over a frictionless pulley at the point, p. Vectors showing the tension in this cable (inclined 30° from the horizontal) and the weight are drawn at p. Find the tension in the supporting cable, mp, and the reaction at n. *Ans.* 1667 lbs., 3055 lbs.

48. Solution by moments. To repeat the second condition of equilibrium: If a set of forces is in equilibrium, the sum of the moments of these forces about any point in their plane equals zero.

Problem 118. In the boom shown in Fig. 64 find the tension, F, in the cable, and the reaction, R, on the boom, by the method of moments, if the 1000-lb. load is applied at point, m.

Solution. The reaction, R, at n is now horizontal. Take moments about the point, p. The moment of F about p is zero since its line of action is through p.

$$R \times 6 = 1000 \times 8, \text{ whence } R = 1333 \text{ lbs.}$$

Take moments about the point, n. The moment arm of F about n is the perpendicular distance from n to mp (shown dotted in Fig. 64). By geometry this perpendicular distance is $8 \times \frac{6}{10} = 4.8$ ft. Then:

$$F \times 4.8 = 1000 \times 8,$$

from which

$$F = 1667 \text{ lbs.}$$

In the following problems find the reactions at m and n by the method of moments.

PROBLEM 119. Fig. 70. *Ans.* 13,700 lbs., 14,900 lbs.
PROBLEM 120. Fig. 71. *Ans.* 4667 lbs., 5090 lbs.
PROBLEM 121. Fig. 77. *Ans.* 73.3 lbs., 89.8 lbs.
PROBLEM 122. Fig. 79. *Ans.* 1667 lbs., 3055 lbs.
PROBLEM 123. Fig. 69. *Ans.* 1910 lbs., 2058 lbs.

PART V

FRAMEWORK—CRANES AND TRUSSES

49. It is often required to find not only the reactions supporting a structure as a whole, but to determine also the forces upon the members of which the structure is composed. For a definite solution the members are assumed to be pin-connected, and for ease in solution the weight of each member itself may sometimes be neglected.

The procedure is, first, find the reactions on the structure as a whole from the given data. Then take one of the members on which one applied force, or reaction, has been determined and analyze for the forces on that member as a whole.

PROBLEM 124. Find all the forces on the mast, boom and strut of the crane shown in Fig. 80, disregarding the weights of the members.

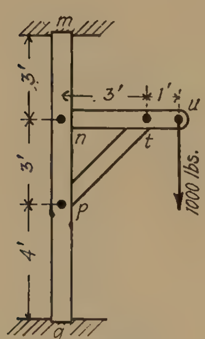


FIG. 80.

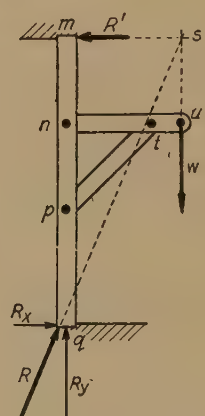


FIG. 81.

Solution. See the space diagram, Fig. 81. Considering the structure as a whole, it must be held in equilibrium by three forces: R', which must be horizontal at m; gravity force on the load, 1000 lbs.; and a reaction R, at q. The lines of

action of these three forces must meet at a point (Art. 44) and since the point is the intersection, s, of the 1000-lb. force and R', this locates the line of action of R as shown. Taking moments about q:

$$R' \times 10 = 1000 \times 4,$$

from which $$R' = 400 \text{ lbs.}$$

To find R, take the components of R as shown in Fig. 81. Then

$$R_x = R' = 400 \text{ lbs., and } R_y = 1000 \text{ lbs.}$$

from which $$R = \sqrt{(1000)^2 + (400)^2} = 1080 \text{ lbs.}$$

Forces on the boom. Since there are only two points, n and t, Fig. 80, at which the boom is supported, the reactions there, together with the 1000-lb. load, make the boom a three-force member. The space diagram for the boom is shown in Fig. 82. The force from the strut to the boom, by inspection, acts upward; and its line of action must be inclined from the vertical to the same degree as the strut, that is, 45°, since the strut is a two-force member (Art. 42). The intersection, o, Fig. 82, determines the line of action of the force through n, which, by inspection,

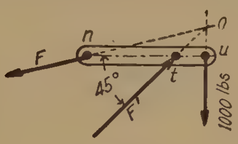


FIG. 82.

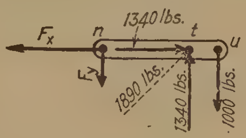


FIG. 83.

must be downward. To find the force through t the method of moments is most convenient, using n as the center of moments. The perpendicular from the line, ot, Fig. 82, is $0.707 \times 3 = 2.12$ ft. Hence the force through t equals $1000 \times 4 \div 2.12 = 1890$ lbs. To find the force through n the component method will be used. Referring to the force through t, 1890 lbs., the horizontal component is:

$$0.707 \times 1890 = 1340 \text{ lbs.},$$

which is also the vertical component. Referring to Fig. 83, F_x and F_y are the components of the force through n, and, from the figure:

$$F_x = 1340 \text{ lbs., and } F_y + 1000 = 1340,$$

from which $$F_y = 340 \text{ lbs.}$$

The force through n then is:

$$\sqrt{(1340)^2 + (340)^2} = 1380 \text{ lbs.},$$

at an angle $$\tan^{-1} = \frac{340}{1340} = 14.25°$$

with the horizontal.

Forces on the strut. The force from the strut to the boom, Fig. 82, was found to be 1890 lbs. Consequently the force from the boom on the strut, at t, is the same amount, as shown in Fig. 84, but opposite in direction. Since the strut is a two-force member the force from the mast on the strut, at p, is colinear with and equal to but opposite in direction to the force on the strut at t, as is shown in Fig. 84.

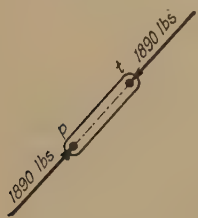


Fig. 84.

Forces on the mast, are shown in Fig. 85 and are all determined in the preceding section.

Problem 125. Using the values of forces and angles given in Fig. 85 check the solution of Problem 124 by showing that the mast is in equilibrium according to conditions (c) and (d), Art. 44.

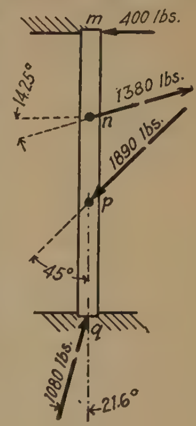


Fig. 85.

In the following problems, numbers 126–129 inclusive, using the method of similar triangles, moments or components, find the forces on all the members of each structure, and show all space diagrams as was done in Problem 124.

Problem 126. Fig. 86. *Ans.* 1237, 2069, 1905, 2475 lbs.
Problem 127. Fig. 87. *Ans.* 5333, 5693, 4740, 5340 lbs.

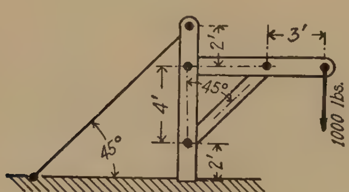


Fig. 86.

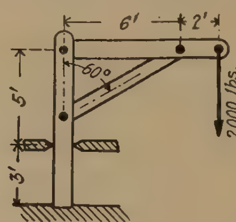


Fig. 87.

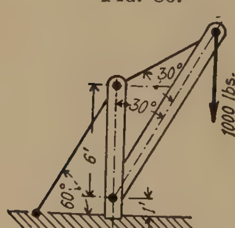


Fig. 88.

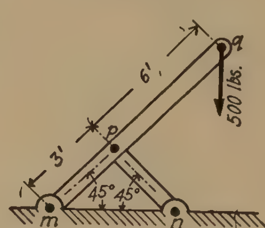


Fig. 89.

Problem 128. Fig. 88. *Ans.* 1485, 2400, 1000, 1733 lbs.
Problem 129. Fig. 89. *Ans.* 792, 1060 lbs.

50. Structures, weights of members considered. Members under two forces only, neglecting weight, become three-force members when the weight is taken into account. The end reactions cannot be colinear, as incorrectly shown by Fig. 90 (except when the member is vertical), since the end reac-

FIG. 90.

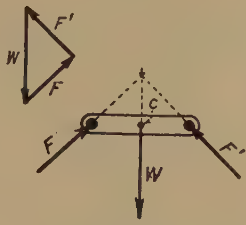


FIG. 91.

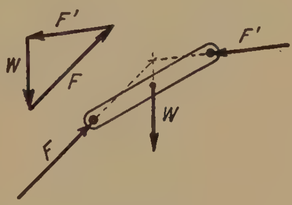


FIG. 92.

tions must have vertical components to balance the weight. The correct space diagram and force triangle are shown in Fig. 91 for a horizontal member and in Fig. 92 for an inclined one. The solution follows the same lines as before.

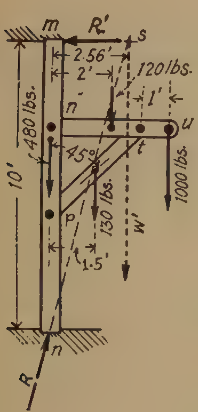


FIG. 93.

PROBLEM 130. Find the forces on the mast, strut and boom in magnitude and direction, of the structure shown in Fig. 80 if the weight of the mast is 480 lbs., of the boom 120 lbs. and of the strut 130 lbs.

Solution. Fig. 93 shows the space diagram of the structure as a whole. As will be noted the reaction, R, meets the reaction, R', which is horizontal, and the resultant, W', of the several gravity forces, at the point, s. The force diagram is drawn in Fig. 94.

If the method of moments is chosen for finding the magnitude of the reaction, R', we have, taking moments about n:

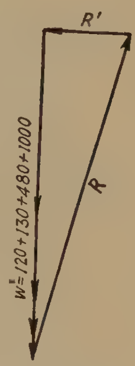


FIG. 94.

$$120 \times 2 + 130 \times 1.5 + 1000 \times 4 = R' \times 10,$$

whence

$$R' = 443.5 \text{ lbs.}$$

and the method of components for the reaction, R,

$$R = \sqrt{(1000 + 480 + 130 + 120)^2 + (443.5)^2} = 1750 \text{ lbs.}$$

The angle made by the line of action of R' with the vertical equals

$$\sin^{-1}\frac{443.5}{1750} = 14.3°.$$

Fig. 95 is the space diagram of the boom showing the components of the unknown forces at n and t.

Fig. 96 is a similar diagram for the strut. It is to be noted that F'_x and F_y' are equal and opposite to F_x'' and F_y'' respectively.

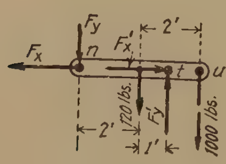


FIG. 95.

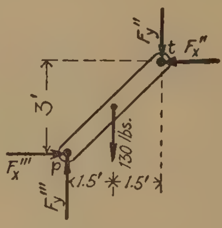


FIG. 96.

For the boom:

$$F_y + 120 + 1000 = F_y', \quad \text{and} \quad F_x = F_x'.$$

Moments about n: $$F_y' \times 3 = 120 \times 2 + 1000 \times 4,$$

whence $$F_y' = 1413 \text{ lbs.}$$

Inserting this in the y-component equation, and solving for F_y

$$F_y = 1413 - 120 - 1000 = 293 \text{ lbs.}$$

For the strut:

$$F_y''' = 130 + F_y'', \quad \text{and} \quad F_x''' = F_x''.$$

Since $$F_y'' = F_y' = 1413, \quad F_y''' = 130 + 1413 = 1543 \text{ lbs.}$$

Moments about p: $$F_x'' \times 3 = 130 \times 1.5 + 1413 \times 3,$$

whence $$F_x'' = 1478 \text{ lbs.} \quad (= F_x''' = F'_x = F_x.)$$

Now, combining the components of the various forces on the boom the force $F = \sqrt{(1478)^2 + (293)^2} = 1509$ lbs., at an angle of 11.2° with the horizontal; and F' on the boom $= F''$ on the strut $= \sqrt{(1478)^2 + (1413)^2} = 2043$ lbs. at an angle of 43.7° with the horizontal; and F''' from mast to strut $= \sqrt{(1478)^2 + (1543)^2}$ 2140 lbs., at an angle of 46.2° with the horizontal.

The forces *on* the mast may now be shown on a space diagram of the mast, as in Fig. 97.

Fig. 98 and Fig. 99 show the forces *on* the boom and *on* the strut respectively.

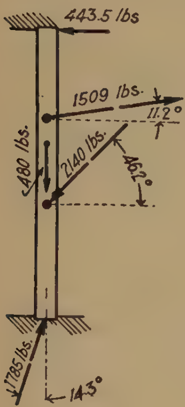


FIG. 97.

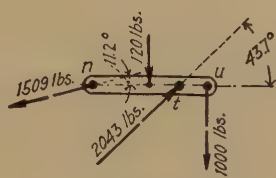


FIG. 98.

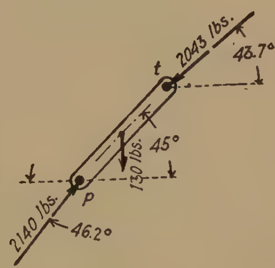


FIG. 99.

PROBLEM 131. If the boom of Fig. 12 weighs 150 lbs. and the weight, W, is 500 lbs., what are the reactions at m and p? *Ans.* 580, 813 lbs.

PROBLEM 132. Find the forces on the pin-connected structure, shown in Fig. 86 if the weights of the mast, boom and strut are 800 lbs., 700 lbs., and 300 lbs., respectively. Neglect the weight of the cable.

Ans. 1775, 4250, 2600, 3660, 3590 lbs.

PROBLEM 133. The boom shown in Fig. 79 has a weight of 800 lbs. Neglecting the weight of the cable what are the magnitudes of the reactions?

Ans. 2335, 3760 lbs.

PROBLEM 134. What are the reactions of the pin-connected structure shown in Fig. 89 if the members forming it weigh 100 lbs. per foot of length?

Ans. 1501, 2230 lbs.

51. Theorem. The polygon of forces. *If three or more coplanar forces whose lines of action meet in a common point are represented by their vectors laid down in order as the sides of a polygon, then the line drawn from the starting point of the first vector to the final point of the last vector represents the resultant in magnitude and direction.* The closing line is opposite in order of direction to the other vectors. If the initial point coincides with the final point, the resultant is zero, the forces are in equilibrium and all vectors are arranged in order of direction.

Given the forces shown in Fig. 100. The resultant of F and F' is F_r', Fig. 101; that of F_r' and F'' is F_r''; that of F_r'' and F''' is F_r. F_r is therefore the resultant of the several forces. The polygon could have been

drawn omitting F_r' and F_r'', in accordance with the above theorem. It is apparent that the polygon of forces is simply an extension of the triangle of forces. It, also, is often referred to as the force diagram.

If F_r, in Fig. 101, is reversed in direction, and added in Fig. 100, at the

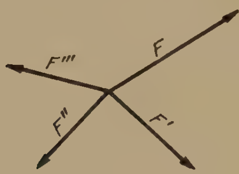


Fig. 100.

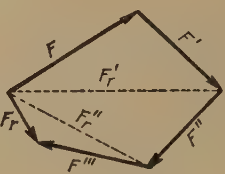


Fig. 101.

common point of intersection, it becomes an equilibrant, F_e. The force polygon, Fig. 101, would then be closed by F_e pointing oppositely to F_r.

Problem 135. Assume that F and F' in the system of forces given in Fig. 75 are 200 lbs. and 150 lbs. respectively. Using a scale of 1 in. = 50 lbs. lay out a force polygon for the forces shown and find the resultant in magnitude and direction. Show this resultant, dotted, on the original figure.

(Note, that the same value results, no matter in what order the vectors are arranged).

Ans. 120 lbs. to the left at 63.5° with the horizontal.

If the lines of action of the coplanar forces applied to a body do not meet in a common point, then:

(*a*) If the forces are in equilibrium the force polygon determines two unknown forces in magnitude, if their directions are known; or it determines one unknown force in magnitude *and* direction; but in neither case does it *locate* the unknowns.

(*b*) If the force polygon closes, it does not follow that equilibrium exists. For example, Fig. 102 represents a one-foot square board, with equal forces, F, of 10 lbs. each applied at the corners as shown. The corresponding force polygon would close, but clearly equilibrium does not exist because the board is under the action of two clockwise couples of 10 pound-feet each, or an unbalanced turning moment of 20 pound-feet.

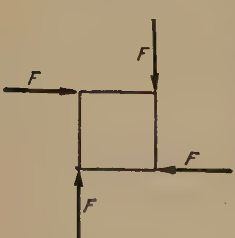


Fig. 102.

(*c*) If the applied forces are parallel and either in equilibrium or unbalanced, the force polygon consists of colinear vectors. It is then possible to determine only one unknown in magnitude, but not in location.

An extension of the force polygon is therefore needed for graphic solution of such problems.

52. **The funicular polygon.** Let us take the same forces as represented by Fig. 100, with the same directions, but located as shown by Fig. 103. It is required to find their resultant in magnitude, direction and location.

The force polygon is constructed as though the forces in the space diagram met in a common point, and is the same as Fig. 101. This is again shown by Fig. 104, with the corners of the polygon designated by numerals, in order, beginning at the starting point of the first vector drawn. Additional construction is shown as follows:

FIG. 103.

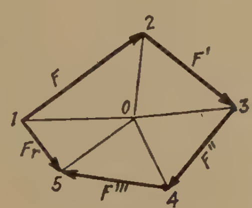


FIG. 104.

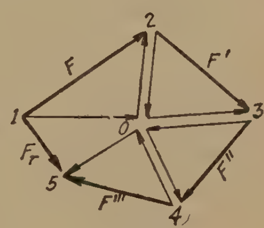


FIG. 105.

Point 0, called the pole, is chosen at random within the force polygon. From it are drawn lines (called rays) to the corners of the polygon, 0–1, 0–2, 0–3, etc. Each adjacent pair of rays, with the opposite vector, may be considered a force triangle. Thus, vector F (1–2) is resolved into oblique components 0–1, and 0–2; vector F' (2–3) is resolved into two components, 2–0 and 0–3, of which 0–2 is the same as for F in magnitude and location, but opposite in direction, and so on. Figure 105 demonstrates this more clearly by showing each ray as two parallel vectors instead of superimposed. Component 0–2 for F balances 2–0 for F'; and so on around, leaving 1–0 and 5–0 as the only unbalanced components. The resultant of these unbalanced components is F_r. Therefore F_r is the resultant of all the forces, as in Art. 51.

Let us now apply the construction to determine the location of F_r. Figure 106 shows the space diagram for the given forces, F, F', F'', etc., as in Fig. 103, except that the vectors for the forces have been moved, at convenience, along their respective lines of action. In Fig. 106, select a point, 1, at convenience, on the line of action of F. Through 1 draw a line parallel to ray 1–0, in the force polygon, and another line parallel to ray 0–2. On these two lines is constructed the parallelogram of forces for F, correspond-

ing to the triangle of forces, 1–0–2, in Fig. 104. Continue the line drawn in Fig. 106 parallel to ray 0–2 until it intersects the line of action of F' at point 2, and on its continuation construct the parallelogram of forces for F' noting in Fig. 106, that the colinear components of F and F' are equal and opposite, and therefore balance.

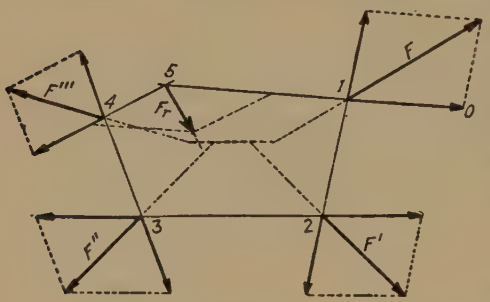


Fig. 106.

Following this construction for the remaining forces, it is seen that one component of F'' balances the remaining one of F', and the other component of F'' is balanced by a component of F'''. This leaves one unbalanced component of F''' and one of F. Continue the lines of action of these components until they meet at point 5, and find their resultant, F_r, as indicated. This locates the line of action of F_r.

The preceding construction may seem laborious, but it has many applications, and the labor may be considerably reduced. It is not necessary to lay out each parallelogram of forces. A simplified construction is shown by Fig. 107, which starts as a space diagram of the given forces. Taking this in connection with Fig. 104, the following construction is used:

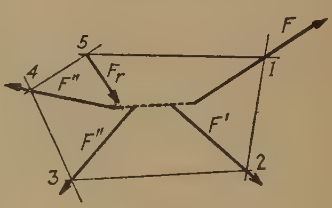


Fig. 107.

Between the lines of action of F and F' in the space diagram (Fig. 107) draw a line 1–2, parallel to the ray 0–2 between F and F' in the force polygon (Fig. 104). At point 2, in the space diagram draw a line, 2–3, between the lines of action of F' and F'', parallel to the ray 0–3 between F' and F'' in the force polygon.

Construct line 3–4 in the space diagram similarly.

The intersection of a line through 4 (parallel to ray 0–5) in the space diagram, with a line through 1 (parallel to ray 0–1) gives a point, 5, on the line of action of F_r. Through 5, draw vector F_r equal and parallel to F_r as found in the force polygon.

In Fig. 106 or 107, the polygon 1–2–3–4–5 is called the *funicular polygon*, and its sides are called *strings*.

PROBLEM 136. Take F and F' in Fig. 75 as 200 lbs. and 150 lbs. respectively. Show a space diagram with these forces acting on a plank: the 220-lb. force acting at the left-hand end of the plank; the 125-lb. force with its point of application 1 ft. from the left-hand end; F with its point of application 2 ft. from the left-hand end; F', 3 ft.; and the 100-lb. force $1\frac{1}{2}$ ft. Use a vector scale of 1 in. = 100 lbs. and a space scale of 1 in. = 1 ft. Draw the force and funicular polygons, and find the resultant of all the forces in magnitude, direction and location. How many feet from the left-hand end of the plank does the line of action of the resultant intersect it? *Ans.* 120 lbs. 63.5° from horizontal, 0.7 ft.

***53*. Conditions of equilibrium.** In the preceding article a *resultant* of a set of forces was found graphically. If this resultant is reversed as shown by F_e in Fig. 108 (disregard for the present, the dotted vectors), the set of forces is in equilibrium, and a force polygon drawn for the set, including F_e, would close with the vectors in directional order. Let there be added to the set, a couple indicated by the dotted vectors of Fig. 108. The force polygon would again close having added to it two parallel equal

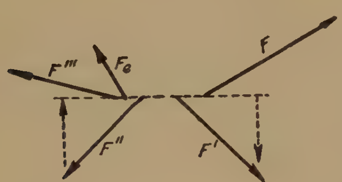


FIG. 108.

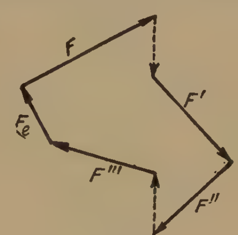


FIG. 109.

sides, and would be in directional order. This is shown by Fig. 109. But the forces are *not* in equilibrium since all are balanced except those of the couple. Consequently the polygon of forces is not the only criterion of equilibrium, when the given set of forces has not a common point of application.

If a funicular polygon be made for Fig. 108 including the couple, it will be found that the first and last strings do not meet at the starting point of the first string drawn. This is true because the strings represent balancing components of forces in equilibrium; if the given forces are not in equilibrium the components will not balance. Therefore,

If a set of coplanar forces holds a body in equilibrium, and does not act at a common point on the body, (*a*) the force polygon *must* close and (*b*) the funicular polygon *must* close.

PROBLEM 137. Using the data and construction for Problem 136, and taking the plank as 3 ft. long, add the equilibrant, F_e, and a clockwise couple of 150 lb.-ft.

with one force of the couple acting at one end of the plank, and the other force at the opposite end, draw the force polygon for this new set of forces. Draw the funicular polygon and show that it does not close.

54. **The force and funicular polygons for coplanar parallel forces.** For this case, the force polygon consists of colinear vectors, and will be referred to as the force diagram. The pole, 0, from which the rays are drawn must be chosen at one side of the force diagram which is a straight line. Thus, given a simple beam, Fig. 110, loaded as shown; it is required to find, graphically, the magnitude of the reactions. A modified form of Bow's notation (See Art. 55) will be used to simplify the construction. This is modified to adhere to the notation elsewhere used in this work. Beginning with any pair of vectors on the space diagram, Fig. 110, as F and R, put the

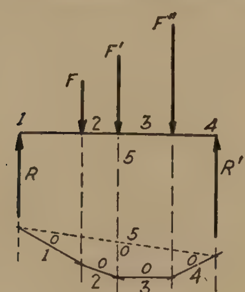


Fig. 110.

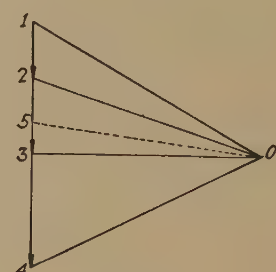


Fig. 111.

numeral, 1, between them; 2 between the next pair; and so on. Then the force, F, is read 1–2, after the numerals on each side of the vector for F; F' is read 2–3, F'' is read 3–4, R' is read 4–5 and R is read 5–1. Lay out the applied forces as a force diagram as shown by Fig. 111. Arrow heads need not be used, as the sequence of the numerals indicates direction. Since R and R' balance the given forces, R' must be written 4–5 in the force diagram, and R, 5–1, both pointing upward; but the location of point 5 is not yet known. Select point 0, as the pole in Fig. 111, and draw the rays. Draw the funicular polygon in the space diagram (Fig. 110) by making the string between the lines of action of force 1–2 and 5–1 parallel to ray 0–1; the string between forces 1–2 and 2–3, parallel to ray 0–2; the string between forces 3–4 and 4–5 parallel to 0–4. (Note in each case, that the number of the ray fixes the parallel string as between the two forces having the same numeral in common.) Now, close the funicular polygon by the dotted line between R and R', then draw the missing ray, 0–5 (shown

dotted), parallel to the last drawn string. This fixes point 5, on the force diagram, and R equals vector 5–1 in scale units, and R' equals vector 4–5 in Fig. 111.

PROBLEM 138. In Fig. 110, if F is 80 lbs. and 1 ft. from the left-hand support; F', 100 lbs. and 1.5 ft. from left-hand support; F'', is 150 lb. and 4 ft.; and the right-hand support is 6 ft. from the left-hand support, find R and R' by preceding method. *Ans.* 190 lbs.; 140 lbs.

55. **Bow's notation** differs from that given in the preceding article in that lower-case letters are used instead of numerals in the spaces between the forces on the space diagram, and the forces, F, F', etc. (Fig. 110), are called ab, bc, etc. Corresponding upper-case letters are used to designate the vectors in the force diagram, as AB in Fig. 111, instead of 1–2. The strings of the funicular polygon may be referred to by the same letters as on the corresponding rays, but in lower case. Thus ray AO is parallel to string ao.

56. **Graphical analysis of trusses for roofs and bridges.** In this type of problem, it is necessary to determine the reactions at the supports, and the internal stresses of the truss members, due to the loading under service.

A roof truss is subjected to the weight of the roofing laid at right angles to the truss, to an estimated snow load which merely adds to the weight per linear foot of truss due to roofing, and a wind load which is assumed at maximum value at right angles to the roofing. There may also be dead loads on the under members of the truss, such as due to a travelling crane on an attached runway.

A bridge truss is subjected to its own weight and other dead loads, and the live load of traversing trains or traffic. These may all be estimated in pounds per lineal foot of bridge, depending upon conditions.

When the forces are all vertical the reactions at the supports may be found by the method of Art. 54, or, if symmetrically loaded, be taken each as half the total load. It then remains to find the internal stresses of the truss members, which are considered as pin-connected.

PROBLEM 139. Consider the roof truss of Fig. 112 which is subjected to a load of 3200 lbs., and freely supported at its ends. The reactions are then 1600 lbs. each. Find the stresses in the members.

Solution. The vectors over the truss represent the concentrated forces over each joint equivalent to uniform loading when the panels between the joints are equal in length. The spaces between the lines of action of the external forces and the spaces included by the truss members in the space diagram are numbered as

shown. The force polygon at the left-hand support is represented by Fig. 113, the vector 1–2 being equal to 1600 lbs. and 2–3 to 400 lbs. Vectors 3–8 and 8–1 represent the forces exerted by the upper and lower members of the truss at the left support, and equal 2400 lbs. and 2080 lbs. respectively, as found by scale. Force 3–8 in the space diagram acts to the left downward, and 8–1 to the right, since the order

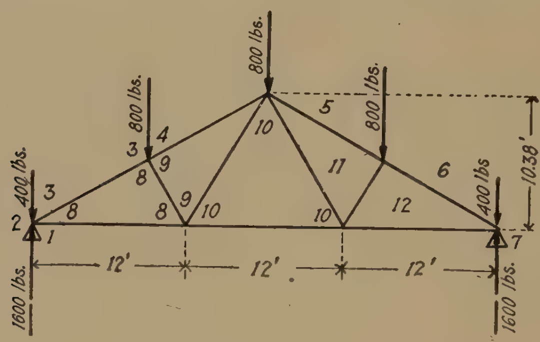


Fig. 112.

of the numerals in the force polygon indicates the direction of vectors represented. Hence the upper member of the truss is in compression, and the lower in tension.

Take next, the middle top joint on the left (Fig. 112), which will be referred to as joint 3–4–9–8. The force 8–3 is opposite in direction, and equal to force 3–8, on the joint 1–2–3–8 (disregarding weight of member). The force diagram for joint

Fig. 113.

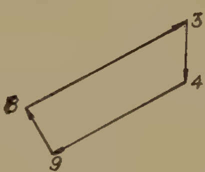


Fig. 114.

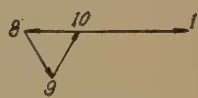


Fig. 115.

3–4–9–8 is shown by Fig. 114, the known vectors being 8–3 and 3–4. By scaling for the unknowns, 4–9 equals 2000 lbs. and 9–8 equals 690 lbs.; and, by inspection of the force polygon with the space diagram, both members are in compression.

Fig. 115 is the force polygon for the joint 1–8–9–10 constructed on the two known vectors 1–8 and 8–9. The force 9–10 (690 lbs.) and 10–1 (1390 lbs.) are thus determined, both being tension.

All the forces to the left of the peak are now found. Since the loading is symmetrical, the stresses in the members on the right equal those on the left, correspondingly.

Figure 116 shows a combined force polygon. Vector 3–8, from Fig. 113 becomes 8–3 and is used, with 3–4, to obtain the same polygon, 3–4–9–8, as Fig. 114. The polygon of Fig. 115 is likewise duplicated by corresponding numerals in Fig. 116. The procedure then reduces to laying out the space diagram of the truss and drawing parallel lines as in Fig. 116, to scale. There are certain exceptions which cannot be solved so directly and which will not be treated in this text.

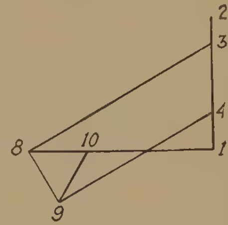


FIG. 116.

The following problems refer to Fig. 112.

PROBLEM 140. Draw the force polygon for joint 4–5–11–10–9, showing closure. Number each vertex of force polygon to conform with Fig. 112. (Note that the stressing to left and right of the peak is symmetrical with reference to a vertical center line.)

PROBLEM 141. Redraw Fig. 116 and supply arrow heads. Place corresponding arrow-heads on all members of the truss in Fig. 112.

PROBLEM 142. If there is no vertical load at joint 5–6–12–11, what is the force 12–11? Why? *Ans.* Zero.

PROBLEM 143. Using the condition of and answer to Prob. 142, what is the force 10–11? Why? *Ans.* Zero.

PROBLEM 144. Remove the 800- and 400-lb. load on the right of the roof, and make the peak load 400 lbs. instead of 800. This leaves the left-hand side of the truss loaded with 400 lbs. over the left-hand support, 400 lbs. at the peak, and 800 lbs. at joint, 3–4–9–8. Find stresses in all members.

Ans. 1600, 1390, 1210, 695, 695, 695, 800, 695, 800, 0, 0 lbs.

PROBLEM 145. Add a 1000-lb. load to each of the joints 1–8–9–10 and 1–10–11–12 of the truss loaded as shown in Fig. 112. Find stresses in all members.

Ans. Symmetrical, 4400, 3820, 4000, 695, 1870, 2520 lbs.

The following problems refer to the figures named.

PROBLEM 146. Fig. 76. Place a vertical support, pin-connected under the peak p. The reactions at m and n are vertical. Find the stresses in all members.

Ans. 715, 570, 0, 810, 570 lbs.

PROBLEM 147. Fig. 117. Find stresses in all members.

Ans. Symmetrical, 14,500, 10,650, 11,100, 3550, 5450, 6400 lbs.

PROBLEM 148. Fig. 118. Find stresses in all members.

Ans. Symmetrical, 5800, 2900, 2900, 0 lbs.

PROBLEM 149. Fig. 119. Can the members, mn and pq, be tie rods taking tension only when a 9000-lb. load is supported at m? When the 9000-lb. load is shifted to n? What are the stresses in those members?

Ans. No, 0, 4250 lbs. compression: Yes, 0, 4250 lbs. tension.

The following problems include the graphic determination of reac-

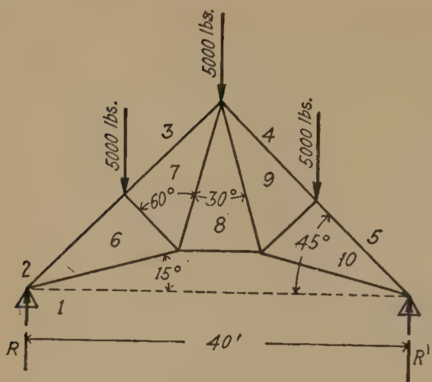


Fig. 117.

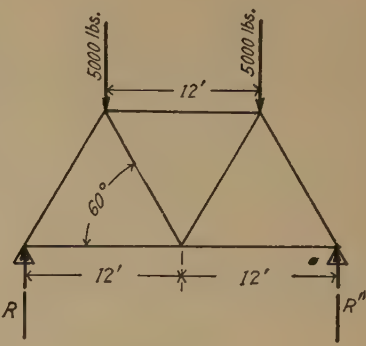


Fig. 118.

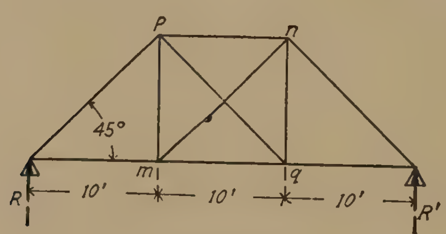


Fig. 119.

tions at supports, when the direction of one of the reactions is not known.

Problem 150. Referring to Fig. 112, assume the left-hand end of the truss to be fixed in brick work, and the right-hand support to consist of rollers, so that the reaction at the right is vertical. Assume a uniformly distributed wind load on the right side of the roof, totalling 900 lbs., at right angles to the roof. Determine the reactions.

Solution. Fig. 120 shows the space diagram in which the dead load is taken at the peak of the truss, and the wind load concentrated at the middle of the right side, and at right angles to it. The effect, in so far as the reactions are concerned, is the same as uniform distribution.

Begin by placing numerals in the spaces between the forces, beginning at the left of the first known force and proceeding clockwise. Force 1–2 equals 3200 lbs., 2–3 equals 900 lbs., 3–4 is vertical and unknown in magnitude, 4–1 is unknown in direction and magnitude.

Fig. 121 shows the force polygon for the truss as a whole, in which the vertex 4 is not located. Hence a funicular polygon must be drawn. Choose a pole, 0, and

draw rays to 1, 2, and 3. The construction of the funicular polygon requires that a string, 0–1, Fig. 120, be drawn parallel to ray 0–1, Fig. 121, *from any point on the line of action of* 4–1 to the line of action of the next force, 1–2, on the space diagram. Since the *only point known* on the line of action of 4–1 *is the point of support* at the left-hand end of the truss, the first string 0–1, must be started there, as shown. The strings 0–2, and 0–3, are then drawn in, parallel to the corresponding rays. Since equilibrium exists, the funicular polygon must close, so the string 0–4 is supplied

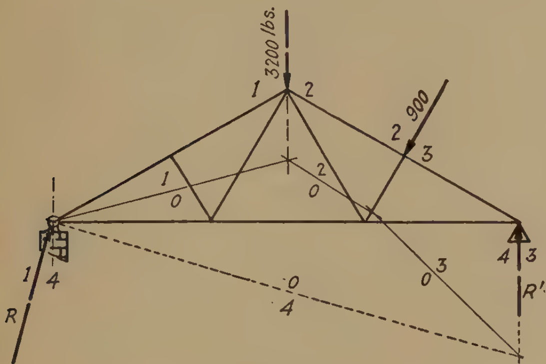


FIG. 120.

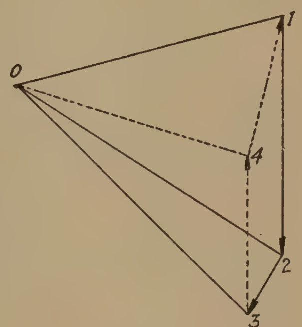


FIG. 121.

(Fig. 120). Parallel to this in the force polygon, the missing ray is drawn, 0–4, which fixes vertex 4. By scaling Fig. 121, reactions 4–1 (1900 lbs.) and 3–4 (2120 lbs.) are found; the first in direction and magnitude, and the second in magnitude only, its direction being known.

PROBLEM 151. Using data of Problem 150 (excepting loads), with dead loads as shown in Fig. 112, and the wind loads distributed with 225 lbs. at the peak and right support and 450 lbs. at the middle, find stresses in all members.

Ans. Left Reaction, 1900 lbs. 14.3° with vertical; Right Reaction, 2120 lbs.

PROBLEM 152. Using data of Problem 151, except that the wind load is on the left side of the roof; the right support consisting of rollers as before: Find the reactions at supports, and the stresses in all members.

Ans. Left Reaction, 2200 lbs. 12.5° with vertical; Right Reaction, 1800 lbs.

PROBLEM 153. Fig. 117. Take the left-hand reaction as vertical, and the right-hand inclined an unknown amount. Apply a wind load at 1000 lbs. at the left support, 2000 lbs. at joint 2–3–7–6 and 1000 lbs. at peak, all at right angles to left-hand side of roof; dead load being the same as in the figure. Find reactions at supports and stresses in all members.

Ans. Left Reaction, 8900 lbs.; Right Reaction, 9040 lbs. at 17.5° with the vertical.

57. Analysis of stresses in truss members by method of sections. The procedure depends upon isolating each joint as though the members at that joint were cut. The imagined "section" thus made is kept in equi-

librium by applying forces at each cut of a member, in sufficient amount as determined by method of components, moments, or similar triangles. The direction of the force at each cut is found by inspection, from which is determined whether the corresponding member is in tension or compression.

PROBLEM 154. Find the stresses in each member of truss of Fig. 112 by method of sections.

Solution. Fig. 122 shows a section cut to the right of the left-hand support as shown by the dotted line. To keep the section in equilibrium it will not suffice to apply a force at the end of member 1–8 and no force at end of member 3–8, because there would be no other horizontal force to balance it. The same thing is true of member 3–8. Therefore, forces must be applied at the cut end of each member.

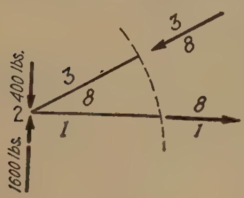


FIG. 122.

Vector 1–2 is in excess of 2–3, so there must be applied a force with a downward component to balance this excess. This can have a line of action only along member 3–8, and is indicated by vector 3–8, which shows the member to be in compression. Vector 3–8 has a horizontal component to the left which can be balanced only by a horizontal force to the right at the cut of member 1–8. This force is shown by vector 1–8, and the lower member is in tension.

From the dimensions of Fig. 112 the angle between the upper and lower members of Fig. 122 is 30°. From Fig. 122

$$\text{Force } 3\text{–}8 \times \sin 30° = 1600\text{–}400$$
$$\text{Force } 3\text{–}8 = 2400 \text{ lbs.}$$
and
$$\text{Force } 1\text{–}8 = 2400 \text{ lbs.} \times \cos 30° = 2080 \text{ lbs.}$$

Fig. 123 shows a section taken to determine forces at joint 3–4–9–8, of which two are known, namely, 3–4 equal to 800 lbs. as given, and 8–3 equal to 2400 lbs. as determined at left-hand support. If moments are taken about the left-hand point of support, the moment of the forces 8–1 and 4–9 are each zero. Therefore, moment of 3–4 equals moment of 9–8 or

$$\text{Force } 9\text{–}8 \times 10.39 \text{ ft.} = 800 \text{ lbs.} \times 9 \text{ ft.}$$
$$\text{Force } 9\text{–}8 = 692 \text{ lbs.}$$

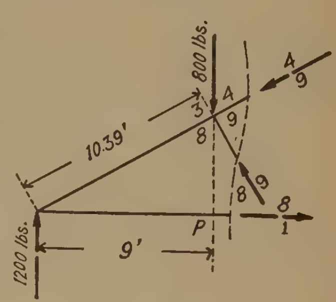


FIG. 123.

and the member 9–8 is in compression.

To find the direction of force 4–9, take as center of moments the point, *p*, which is the intersection of vectors 3–4 and 1–8. The section is now in equilibrium under the moments of the 1200-lb. force, the force, 4–9, and the force, 9–8. The moment

of the 1200-lb. force (clockwise) is greater than that of the force, 9–8 (anti-clockwise) since it has a greater magnitude and a greater arm length. Hence there is an excess towards clockwise rotation around point p, which must be balanced by an anti-clockwise moment from force 4–9. Therefore, 4–9 points downward, as shown, and the member, 4–9, is in compression. The magnitude of 4–9 may be obtained by taking moments about p and equals 2000 lbs.

Fig. 124 shows another section for the same joint, in which the previously determined stress in member 3–8 is used. The solution is best accomplished by taking x and y components along an x-axis colinear with the 2400 and 4–9 vectors.

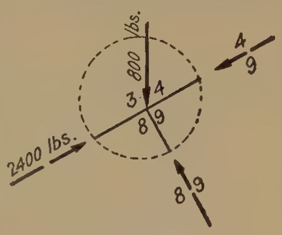


FIG. 124.

In the following problems, find stresses in members by the method of sections.

PROBLEM 155. Fig. 112, for joint 1–8–9–10, without reference to analysis of joint 3–4–9–8.
PROBLEM 156. Fig. 117.
PROBLEM 157. Fig. 118.
PROBLEM 158. For same data as Problem 144.

58. Structures under non-coplanar forces. Many such problems in statics may be reduced to the same terms as apply to coplanar conditions. The side view of an automobile, for instance, represents a coplanar condition in which two reactions, one at each pair of wheels, are readily found. But the automobile rests on four wheels, to allow for which, the reactions found by a coplanar solution are each divided by two, or if the center of gravity is not on a longitudinal vertical section, the reactions are proportioned between each pair of wheels accordingly.

As a further illustration of the solution of problems involving non-coplanar forces, by reduction to equivalent coplanar conditions, consider the sheer leg crane shown by Figs. 125 and 126, in side and front elevations. The load, W, supported at point, n, and the reaction, R, are in the same plane. Choose this as the xy plane and the origin, o, as indicated. Fig. 125 represents a coplanar space diagram in which nm is supposed to be a single support replacing the legs $m'n$ and $m''n$ in Fig. 126. By any of the coplanar methods R and R' may be readily found in Fig. 125. R' is the equilibrant of W and R. If R' be reversed in direction and applied at point, n, it becomes the resultant, F_r, of W and R, and lies in the plane $m'nm''$.

We now have to deal with the coplanar forces, F_r, and R'' and R''',

the inclined reactions at the legs. Revolve the plane $m'nm''$ into the vertical as shown by Fig. 127, make vector F_r equal to R' as found in space diagram, Fig. 125. The full value of the vectors R'' and R''' may now be

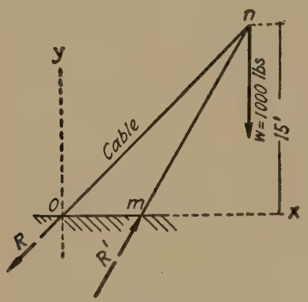


FIG. 125.

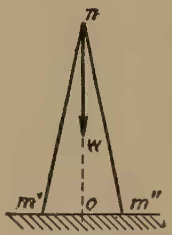


FIG. 126.

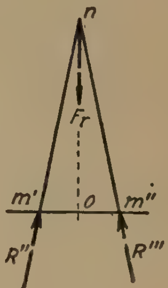


FIG. 127.

found from the space diagram of Fig. 127, by any of the usual coplanar methods.

PROBLEM 159. If in Figs. 125 and 126 point n is 15 ft. from the ground, angle nox is 45°, nmx is 60°, the legs at the ground are 5 ft. apart and W is 1000 lbs., find R'' $(= R''')$. *Ans.* 1390 lbs.

PROBLEM 160. In Fig. 89, imagine the 3-ft. strut to be replaced by two struts at an angle of 60° apart in their own plane. The elevation shown by Fig. 89 remains unchanged. Find the reactions at each of the three points of support. *Ans.* 792, 615, 615 lbs.

Note that the problems of this article deal with forces meeting at a common point.

59. Non-intersecting, non-coplanar forces. Conditions of equilibrium.

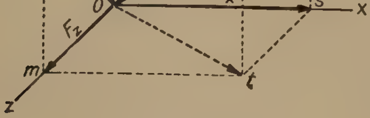


FIG. 128.

Let F in Fig. 128 represent one of the set of such forces. For convenience select an origin, o, at the extremity of vector F. Erect on this vector as a diagonal a rectangular parallelopiped with three sides, os, op, and om colinear with the selected coordinating axes, ox, oy and oz, respectively. Then these sides will represent the x, y, and z components of F. For, in the xy plane, the resultant of vectors op $(=F_y)$ and os $(=F_x)$ is oq; oq, F and om $(=F_z)$ lie in the same plane, and the resultant of oq and om is F. Consequently the rectangular com-

ponents of F are three intersecting sides of the parallelopiped with F as a diagonal.

Vector oq is the projection of F on the xy plane. If F_z equals zero, then F equals its projection on the xy plane, lies in that plane, and has x and y components only. The effect of the z-component is to draw the resultant of F_x and F_y (oq) out on the oy plane, increase its magnitude, and incline it, with reference to the three coordinating axes.

In Fig. 128, vectors oq, ot, and on are the projections of F on the xy, xz, and yz planes, respectively.

Figure 129 shows any force in space referred to the coordinating axes, ox, oy and oz. Continue the line of action of F until it intersects one of the planes, as xz, at point o'. In this plane draw through o', lines $o'x'$ and $o'z'$ parallel to the coordinating axes of the plane; and erect at o' a perpendicular, $o'y'$. Call the angles between F and $o'x'$, $o'y'$ and $o'z'$, β, θ and γ respectively, then,

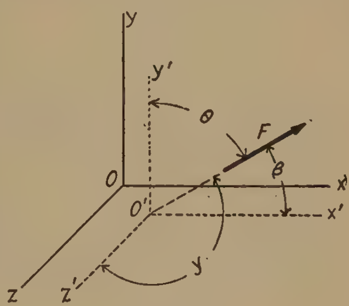


FIG. 129.

$$F_x = F\cos\beta, \quad F_y = F\cos\theta, \quad \text{and} \quad F_z = F\cos\gamma,$$

and

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}.$$

Referring again to Fig. 128, let F be in equilibrium with one or more non-coplanar forces, then:

(*a*) *The components of all the forces in each plane must satisfy coplanar conditions of equilibrium; that is, the sums of the components parallel to each axis of the plane must each equal zero, and the sum of the moments of the components in each plane about the origin equals zero.*

(*b*) *The projection of the forces on each plane must also satisfy coplanar conditions of equilibrium.*

Item (*b*) follows from (*a*), since in any plane the projection of a force is the resultant of (or equivalent to) the components of the force in that plane.

PART VI

FRICTION

60. The laws governing mechanical friction are different according to whether the friction surfaces are perfectly lubricated, imperfectly lubricated or dry. By perfect lubrication is meant that a film of oil separates the bearing surfaces so that they do not actually touch each other anywhere; by imperfect lubrication is meant partial contact and an imperfect or partial film of oil; and by dry friction, a solid contact of surface to surface without an intervening substance.

Friction varies with the following conditions:

(*a*) The velocity of either surface with respect to the other.

(*b*) The magnitude of the normal forces holding the surfaces in contact (friction is generally taken as directly proportional).

(*c*) The nature of the surfaces; that is, the material and its smoothness.

(*d*) The condition of lubrication, perfect, imperfect or dry.

61. Static and Kinetic friction. If the velocity is zero, the friction may have any value from zero to a definite maximum fixed by the other items above, (*b*) to (*d*). In Fig. 130 a body, P, is shown resting on a horizontal plane. If there is no active force, F, applied tending to move it to right or left, there is no friction force. If F is a small force acting to the right, insufficient to cause motion, it must be balanced by friction which will then exist in like amount but acting to the left as shown by the force, $\bar{F}$. If F is increased, $\bar{F}$ will increase (for equilibrium) *until motion impends.* The opposing force, F, as instanced above, up to the limit of impending motion, is called *static friction.*

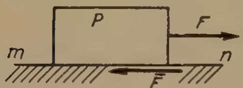


FIG. 130.

Referring again to Fig. 130, if motion is established, experiment shows that the value of F may be reduced very materially to *maintain a constant velocity.* The co-existent and equal value of $\bar{F}$ is called kinetic friction. Its value is fixed by the items (*a*) to (*c*) of Art. 60 for dry and imperfectly lubricated surfaces; and for perfectly lubricated surfaces, by items (*a*) and

(*b*). Note that if the velocity is constant (a condition known as uniform motion, Art. 65) all the conditions of equilibrium apply (Art. 44).

62. Coefficient of friction. Art. 60 (*b*) states that friction between two surfaces is directly proportional to the normal forces holding them in contact. In Fig. 131:

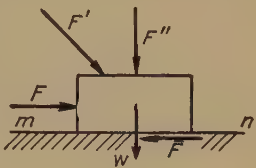


FIG. 131.

Let $\overline{F}$ = the friction force between the two surfaces,

f, f' = a coefficient (referred to later as the "coefficient of friction"); f applying to a condition of rest with motion impending, and f' to uniform motion,

F, F', F'' = forces applied other than gravity,

W = the weight of the body,

$\Sigma(F_y)$ = the sum of the y components of F, F', F'' and W perpendicular to the plane mn, = the total normal force pressing the surfaces together.

Then, if motion impends,

$$\overline{F} = f\Sigma F_y \quad \text{and} \quad f = \overline{F} \div \Sigma F_y,$$

and, if there is uniform velocity,

$$\overline{F} = f' \times \Sigma F_y \quad \text{and} \quad f' = \overline{F} \div \Sigma F_y.$$

PROBLEM 161. If in Fig. 131, F is horizontal and equals 5 lbs., F' is at 30° with the horizontal and equals 10 lbs., F'' is vertical and equals 15 lbs., and the weight of the block is 50 lbs., what is the coefficient of friction if motion is impending and the plane mn is horizontal?

Solution. By components,

$$\overline{F} = F + F' \cos 30,$$

whence

$$\overline{F} = 5 + 10 \times 0.866 = 13.66 \text{ lbs.},$$

also,

$$F_y = 0 + F' \sin 30 + F'' + W,$$
$$= 0 + 10 \times \sin 30 + 15 + 50 = 70 \text{ lbs.},$$

and

$$f = \overline{F} \div \Sigma(F_y) = 13.66 \div 70 = 0.195.$$

PROBLEM 162. Fig. 131. Take $F' = 20$ lbs., F'' equal 30 lbs. and $W = 60$ lbs. If the coefficient of friction, f, is 0.25 what must F be to start motion?

Ans. 7.7 lbs.

PROBLEM 163. If the block shown in Fig. 130 weighs 100 lbs. and the coefficient of friction equals 0.25 and F is 20 lbs., what is the friction force?

PROBLEM 164. If the plane, mn, of Fig. 131 is slanted upward to the right 30°, the vectors F, F' and F'' maintaining the same angles with respect to mn and all other data as in Problem 162, what must F be to start motion up the plane?

Ans. 35.7 lbs.

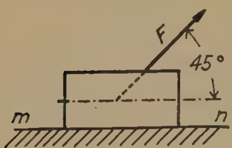


FIG. 132.

PROBLEM 165. If the weight of the block shown in Fig. 132 is 100 lbs. and f is 0.3, what must F be to start motion to the right? *Ans.* 32.6 lbs.

PROBLEM 166. Fig. 132. If f is 0.3, and motion impends, what is the value of W in terms of F?

63. Normal reaction and angle of friction. The sum, $\Sigma(F_y)$, of the components, perpendicular to the plane, of the applied forces and of the weight must be balanced by a perpendicular component of a reaction from the surface upon which the body bears. Thus, in Fig. 133, F_r is the resultant of all the applied forces and of the weight of the body, F_y is the component of F_r perpendicular to the plane, mn, and R_n is a *normal reaction from* mn, balancing F_y. The other component of F_r, marked F_x in Fig. 133, is balanced by the friction force $\overline{F}$. It follows that R_n and $\overline{F}$ are rectangular components of a reaction, R, from the surface mn *to* the body, and that R must be equal to and colinear with F_r, and opposite in direction.

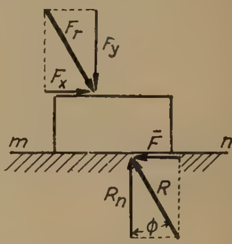


FIG. 133.

In Fig. 133,

$$f = \frac{\overline{F}}{F_y} = \frac{\overline{F}}{R_n},$$

or the coefficient of friction, with motion impending, equals the ratio of the friction force to the normal reaction.

The reaction, R, Fig. 133 (of which $\overline{F}$ and R_n are components) must be slanted with respect to the surface, mn. Calling the angle made between R and a perpendicular to the surface, ϕ, then, from the figure,

$$\text{Tan } \phi = \frac{\overline{F}}{R_n} = f.$$

The angle ϕ is called the angle of friction, and is that angle whose tangent equals the coefficient of friction when motion impends. The same is

true for f', in uniform motion. The friction angle then is called ϕ' and is that angle whose tangent is f'.

The use of this angle enables a space diagram to be made in which R is shown in direction, located by ϕ. The friction force and the normal reaction need not be shown.

PROBLEM 167. Fig. 134. Find F for uniform motion if f' is 0.2, and W is 50 lbs.

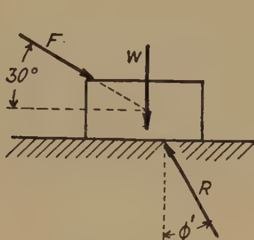


FIG. 134.

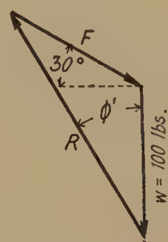


FIG. 135.

Solution. The angle whose tangent is 0.2 equals 11° 20′. Lay off, on the space diagram, vector R inclined 11° 20′ to a perpendicular from plane mn being careful that R points *against* the motion. The corresponding force diagram is shown by Fig. 135, which may be solved for F by scaling.

Attention is called to the fact that when vector F is laid off from the extremity of vector W, Fig. 135, if it has an angle greater than 30° with the horizontal, F is greater and R is greater; and if the angle is less than 30°, F and R are smaller. Therefore, there must be an angle at which F may be applied which gives a minimum value to F.

PROBLEM 168. A body weighing 50 lbs. is to be moved along a horizontal plane, mn, at uniform velocity. If f' is 0.20, what is the magnitude and direction of the *least* force necessary to maintain the motion?

Solution. Fig. 136 shows the space diagram, the line of action of R being fixed by the angle ϕ', whose tangent is 0.20 or $\phi' = 11° 20'$. The line of action F is to be determined. Lay out the force triangle as shown by Fig. 137. The least value of F is obtained when its vector is perpendicular to the vector R. Hence:

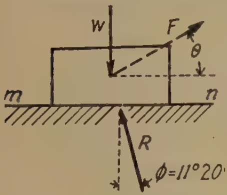


FIG. 136.

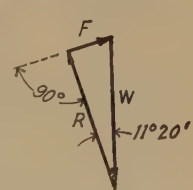


FIG. 137.

$$F = 50 \times \sin 11° 20',$$
$$= 50 \times 0.1965 = 9.82 \text{ lbs.}$$

and the inclination of F, from the horizontal, is, by geometry, 11° 20′.

From this problem it is seen that the least force required to move a body horizontally must be applied upward at an angle from the horizontal equal to the angle of friction.

PROBLEM 169. A body weighing 50 lbs. is to be moved along a plane which slopes upward to the right at an angle of 5° with the horizontal. If f' is 0.20, what is the magnitude and direction of the least force required to move the body uniformly up the plane? *Ans.* 14.1 lbs., 16° 20′ with horizontal.

Most works on elementary mechanics include problems of the type indicated by Fig. 138, representing two wedges, indicated by numerals 2 and 3, with an applied force, F, to lift a load, W. The bottom wedge slides to the right on a horizontal surface of a body, 1, and the top wedge

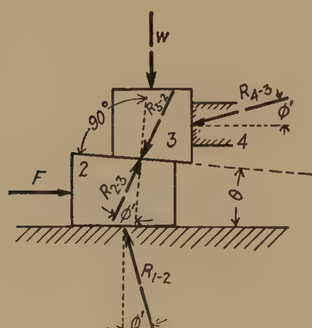


FIG. 138.

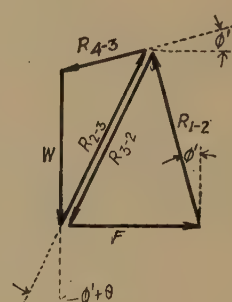


FIG. 139.

slides on the bottom wedge and moves upward sliding also against a vertical stationary surface of body, 4. It should be noticed that if the wedge 2 is considered, the reaction from 1 to 2, indicated by vector R_{1-2}, is against F and the motion of the wedge, 2. The reaction R_{3-2} from wedge 3 to wedge 2, acts similarly, and must point against F and the motion of wedge 2. But if the forces on wedge 3 are analyzed, the reaction, R_{2-3} *from* wedge 2 is equal and opposite to the reaction from wedge 3 to wedge 2. Considering this further, it will be concluded that, as far as the bottom wedge is concerned, friction between the wedges acts to the *left;* but when the forces on the upper wedge are analyzed, friction acts to the *right.* This condition must be thoroughly understood for a successful analysis of such problems.

Figure 139 indicates the graphic solution of the problem of Fig. 138. The directions of all the forces are known since the reactions are inclined to the perpendiculars from friction surfaces by the angle ϕ'. The first

force diagram to be laid off is for the upper wedge, since W is known. F is then determinable.

PROBLEM 170. In Fig. 138, if $W = 1000$ lbs., $\theta = 5°$, and $f' = 0.176$. Find F to cause upward motion of the load. *Ans.* 460 lbs.

Problems of this type are seldom met in Engineering since, for lubricated surfaces, the value of f' is so small that ϕ' approaches zero and the reaction, R, may be taken equal to its normal component, R_n.[1]

In Engineering, dry friction and consequently large values of the coefficients of friction, are practically only of importance in the analysis of forces set up in fasteners (such as a screw thread which closely resembles the problem of Fig. 138) or devices for transmitting forces, such as screw jacks, belt drives, friction clutches, and brakes. Except for the last named, these devices belong more properly to works on Machine Design than to Applied Mechanics, and will therefore not be discussed here, the fundamental principles having been given.

64. Friction of cylindrical surfaces. Since the distribution of the load transmitted from a shaft, or journal, to the bearing supporting it is variable, it is customary to take the friction force as equal to a coefficient times the load in pounds per square inch on the projected area of the bearing, multiplied by the projected area. By "projected area" is meant the length times the diameter of the bearing. As the bearing is always lubricated, the coefficient again is small, and depends upon velocity and pressure as well as upon the methods of lubrication (imperfect or perfect). The problem generally consists of finding the necessary size of bearing for the given conditions. This, also, falls in the province of Machine Design.

[1] Under "Additional Problems," Nos. 6–14 to 6–29, inclusive, are to be found exercises of this kind which may be used as practice in Applied Mechanics.

PART VII

RECTILINEAR AND ROTARY MOTION

65. Velocity. Suppose a body to be in motion, as defined in Art. 4, and that the path described by every point on the body is a straight line. Starting from some datum point, let:

L = the length of the path in feet,
T = the number of seconds required to traverse the path.

Then, if the motion is *uniform*, the velocity, V, is constant, and is designated by

$$V = L \div T.$$

If the velocity varies, its value at any instant is

$$V = dL \div dT.$$

The units of velocity in fundamental equations are "feet per second," or $L \div T$.

According to Art. 4 "motion" involves the relative position of two bodies. For present purposes it is assumed that the body to be moved is one, and the earth, or some body stationary with respect to the earth, is the other.

To determine velocity, *direction* and *magnitude* are required. Velocity is therefore a vector quantity and can be resolved into components, and conversely.

66. Acceleration. When the velocity of a body changes, the body is said to be accelerated. The acceleration, A, is defined as the change in velocity per unit of time. If the acceleration is uniform, and if V' is the change in velocity during T seconds:

$$A = V' \div T,$$

and the acceleration is constant. If the acceleration varies, its value at any instant is

$$A = dV' \div dT.$$

The units of acceleration in fundamental equations are "feet per second per second," or $L \div T^2$.

When the velocity is *increased* the acceleration is plus and is in the same direction as the motion.

When the velocity is *decreased* the acceleration is negative (often referred to as "retardation") and is in a direction opposite to the motion. It follows that acceleration is a vector quantity and can be resolved into components, and conversely.

67. Problems involving distance, time, velocity and **constant acceleration** are readily solved by certain formulas deduced in elementary books on physics. For present purposes these are not necessary, the following method being preferable:

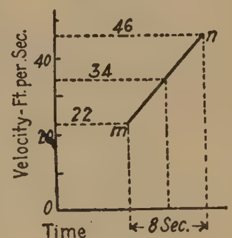


FIG. 140.

Given certain data concerning time, distance, velocity, or acceleration of a body under constant acceleration, to find certain unknowns; a graphic representation is constructed as in Fig. 140. The velocity of the body is plotted vertically, and time horizontally. Point m signifies an initial velocity of 22 feet per second, and point n 46 feet per second, after eight seconds. If the acceleration is constant, the curve between m and n is a straight line, following the law

$$V' = AT,$$

which is the equation of a straight line. The height of the curve at any interval of time indicates the velocity at that time. Obviously the mean height of the line, mn, shown dotted, is the *average* velocity between m and n, or with the figures shown is $(46 + 22) \div 2 = 34$ feet per second.

The acceleration is the vertical distance between m and n divided by the horizontal distance, in scale units. For the data of Fig. 140, $A = (46 - 22) \div 8 = 3$ feet per second per second.

It is sometimes convenient to place under the points on the velocity-time graph, the corresponding distances travelled, but it is important to note that the abscissæ of the diagram are not proportional to distance travelled.

PROBLEM 171. A car has an initial velocity of 22 ft. per sec. If it is uniformly accelerated at a rate of 3 ft. per sec. per sec., what will be its velocity after 8 seconds? How much distance will it have traversed during that time?

Solution. See Fig. 140. Point m represents the initial condition. Increase of velocity = (3 ft. per sec.^{2}) × 8 seconds = 24 ft. per sec. Final velocity = 22 + 24 = 46 ft. per sec., shown by point n. Average velocity = (22 + 46) ÷ 2 = 34 ft. per sec. Distance traversed in 8 seconds = 34 × 8 = 272 ft.

PROBLEM 172. The velocity of an automobile is uniformly increasing from 10 ft. per sec. to 30 ft per sec. in 100 ft. In what additional distance could the velocity be increased to 40 ft. per sec., assuming that the acceleration remains constant?

Solution. See Fig. 141. From the given data locate the initial point, m. The second point, n, is at T seconds and a velocity of 30 ft. per sec.; that is, it is somewhere on the dotted line running through 30 ft. per sec. on the vertical scale and 100 ft. distance from m.

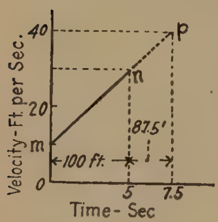


FIG. 141.

Average velocity from m to n = (10 + 30) ÷ 2 = 20 ft. per sec. Therefore, T = 100 ft. ÷ 20 ft. per sec. = 5 seconds which locates n. Between m and n, A = (30 − 10) ÷ 5 sec. = 4 ft. per sec.^{2} At the third point, p, the velocity is given at 40 ft. per sec. Since the acceleration is the same as between m and n, from $A = V' \div T$, the time between n and p is

$$(40 - 30) \div 4 = 2.5 \text{ seconds.}$$

Average velocity between n and p is (40 + 30) ÷ 2 = 35 ft. per sec. and the distance between n and p is

$$35 \text{ ft. per sec.} \times 2.5 \text{ sec.} = 87.5 \text{ ft.}$$

Another method of getting the time between n and p follows from the proportionality of the Fig. 141,

$$\frac{\text{Time } n \text{ to } p}{\text{Time } m \text{ to } n} = \frac{\text{Change of velocity } n \text{ to } p}{\text{Change of velocity } m \text{ to } n}.$$

PROBLEM 173. A railroad train is travelling at the rate of 28 miles per hour. In what distance can it be uniformly brought to rest if it is retarded at the rate of 2 ft. per sec. per sec.? *Ans.* 420 ft.

PROBLEM 174. An automobile with a velocity of 32 miles per hour is uniformly retarded to 22 miles per hour in a distance of one-half a mile. In what additional distance may it be brought to rest at the same rate of retardation? *Ans.* 0.45 mile.

PROBLEM 175. An interurban car starts from rest and is uniformly accelerated to a speed of 30 miles per hour in three minutes. The car runs at this speed for a certain time when the brakes are applied and the car is brought to rest at a uniform rate of retardation in two minutes. If the total distance traversed is 3 miles, what is the total elapsed time? *Ans.* 8.5 minutes.

68. Velocity of points on rotating bodies. If a body has a motion such that every point on it describes a circle, or the arc of a circle, the body is said to be rotating. The datum point, or line, is the common center, or axis of the circular paths.

The linear velocity, V, of any such point is tangential to its path and is equal to the length of its path divided by the time occupied in traversing it. The same relations apply for linear velocity in rotation as in rectilinear motion; the only difference being that the path is the arc of a circle instead of a straight line. If V for any point is constant, the rotation is uniform.

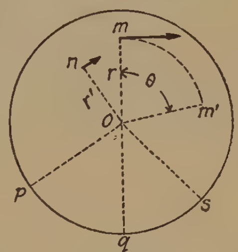


FIG. 142.

It is to be observed, however, that if two points, m and n, on a uniformly rotating body are located at different distances, r and r', from the center of rotation, then the linear velocities of those points are different. In Fig. 142, which represents a disc revolving about the point, o, the length of the path of the point, m, in one revolution of the disc is

$$2 \times 3.14 \times r,$$

and of n is

$$2 \times 3.14 \times r'.$$

Since the time required to complete a revolution is the same for both m and n, their linear velocities are proportional to their radii, r and r'.

PROBLEM 176. A 6-ft. diameter wheel revolving uniformly on its own axis makes 100 turns per minute. What is the linear velocity in feet per second of a point, m, on the rim of the wheel? of a point, n, on the wheel 1 ft. from the center?
Ans. 31.4, 10.5 ft. per sec.

PROBLEM 177. If the speed of rotation of the wheel of Problem 176 is doubled, what are the new linear velocities of m and n?

PROBLEM 178. If the speed of rotation of the wheel of Problem 176 is halved, what are the linear velocities of m and n?

69. A radian is *a plane angle which subtends a circular arc, equal in length to the radius of the arc, and centered at the apex of the angle.*

In Fig. 142, if the arc pq equals its radius, op, in length, angle poq equals one radian. For any angle, as pos,

$$\text{Number of radians} = \frac{\text{length arc } pqs}{\text{length } op},$$

or, calling the angle in radians, θ; the length of the arc, L; and the radius of the arc, r, then

$$\theta = \frac{L}{r}.$$

Note that L and r are both in units of length, and therefore the unit of measurement, the radian, is a pure number.

If the angle subtended is 360 degrees, then

$$L = 2 \times 3.14 \times r,$$

and

$$\theta = \frac{6.28r}{r} = 6.28,$$

that is, there are 2π radians to 360 degrees.

From the relation, $\theta = L \div r$,

$$L = r\theta.$$

That is, the length of a circular arc equals the product of its radius and the subtended angle in radians.

70. Angular displacement. A point on the rotating body, as m in Fig. 142, moves to a position, m', in T seconds, the linear velocity being constant. A line, om, drawn from m to the center of rotation, moves to a position, $m'o$. The angle θ, subtended by mo and $m'o$, is said to be the *angular displacement* of the body.

Angular displacement in fundamental equations is measured in radians. In engineering formulas it is measured in "number of turns" or "revolutions," or fractions thereof. If N' stands for number of turns, the displacement corresponding to N', in radians, is

$$\theta = 2 \times 3.14 \times N'.$$

71. Angular velocity. For uniform motion, *angular velocity* is the angular displacement per unit of time. It is generally signified by ω, in radians per second.

For constant angular velocity:

$$\omega = \theta \div T \text{ in radians per second.}$$

If N'' and N represent number of turns per second, and per minute respectively (N will also be abbreviated " r.p.m."),

$$\omega = 2 \times 3.14 \times N''$$

$$= 2 \times 3.14 \times N \div 60,$$

and

$$N'' = \omega \div (2 \times 3.14),$$

$$N = 60\omega \div (2 \times 3.14).$$

If ω varies, the differential form, $d\theta \div dT$, gives its value at any instant.

72. Relation between angular and linear velocity. Multiplying the equation, $\omega = \theta \div T$, through by r (Fig. 142) gives

$$r\omega = \frac{r\theta}{T}.$$

Since $r\theta = L$ (or circular distance mm', Fig. 142),

$$r\omega = \frac{L}{T} = V,$$

V being the linear velocity of point m in feet per second. Also

$$\omega = \frac{V}{r}.$$

This value for ω is the same as obtained by dividing the linear velocity of point n by r'.

PROBLEM 179. Two wheels, mounted on shafts, are so arranged that one drives the other by friction contact at the circumferences. The driving wheel is 12 ins. in diameter and runs at 525 r.p.m. What is the linear velocity of a point on its circumference? What is the angular velocity?

Ans. 27.45 ft. per sec., 54.9 radians per sec.

PROBLEM 180. Using the data of Problem 179, if the driven wheel diameter is 9 ins., what is its angular velocity? (Assume that there is no slippage between the wheels.)

Ans. 73.2 radians per sec.

PROBLEM 181. An electric motor with an 8-in. pulley drives a line shaft through a 24-in. pulley, belt connected. The linear velocity of a point on the belt is 3000 ft. per min. What are the angular velocities of the two pulleys in radians per second and in r.p.m. if there is no belt slippage?

Ans. 50, 150 radians per sec., 478, 1425 r.p.m.

73. Angular acceleration is the change in angular velocity per unit of time. Exactly parallel equations to those for rectilinear motion apply;

and the relation between angular and linear acceleration is identical to that between the velocities. (Art. 72.)

The units of angular acceleration in fundamental equations are radians per second per second, designated by α. Angular acceleration may also be quoted in terms of change in "revolutions per minute" per minute or per second, and in revolutions per second per second.

Let α be the angular acceleration of a rotating body, corresponding to a change in angular velocity, ω', in the time T; and let r be the distance of any point on the body (not on the axis of rotation) from the axis of rotation, and A the linear acceleration of this point.

Then, if the acceleration is constant,

$$\alpha = \omega' \div T,$$

and if the acceleration varies

$$\alpha = d\omega' \div dT.$$

The relation between the angular acceleration of a body and the linear acceleration of a point on it at a distance r from the center of rotation is

$$r \times \alpha = A.$$

Problem 182. A 12-in. diameter wheel running at 525 r.p.m. is driving a 9-in. diameter wheel by friction contact, no slippage. If the speed of the driving wheel is increased to 600 r.p.m. in 5 seconds, what is the angular acceleration in radians per second per second of each wheel. What is the linear acceleration of a point on the circumference of each?

Ans. 1.57, 2.09 radians per sec.^{2}; 0.785, 0.785 ft. per sec.^{2}

Problem 183. An 8-in. pulley is driving a 24-in. pulley, belt connected, no slippage. The belt speed is 3000 ft. per minute. If the belt speed is increased to 4000 ft. per min. in 8 seconds, what is the angular acceleration of each pulley in radians per second per second? In revolutions per minute per second?

Ans. 6.25, 2.08 radians per sec.^{2}; 59.4, 19.64 r.p.m. per sec.

74. Problems involving displacement, velocity and time, when the acceleration is constant, may readily be solved by the method outlined in Problems 171 and 172.

Problem 184. A cylindrical drum, 3 ft. in diameter, is rotated on its axis by pulling a rope wound around it. If the linear acceleration of a point on the rope is 32.2 ft. per sec. per sec., what will be the revolutions per minute of the drum, if started from rest, at the end of 6 seconds? How many turns will it have made?

Ans. 61.4 turns, 1230 r.p.m.

Problem 185. The revolutions per minute of a flywheel are uniformly decreased from 120 to 80 in 60 turns. In how many additional turns could the speed be decreased to 40 turns per minute, taking the retardation as constant?

Ans. 36 turns.

PART VIII

UNITS OF MEASUREMENT OF FORCE AND MASS

75. Newton's three laws of motion, for the purposes of this book may be stated as follows:

1. A body remains at rest or in uniform motion in a straight line unless acted upon by an unbalanced resultant force.

2. A particle acted upon by a resultant force receives an acceleration in the direction of the force which is proportional to the force and inversely proportional to the mass of the particle. (See Art. 133.)

3. For every active force upon a body there is an equal, opposite and colinear reaction.

76. The standard pound, measurement of force. The unit of force in the English system of measurement, as stated in Art. 2, is the "pound." The basis of this unit is as follows: The French Republic established a standard in concrete form, consisting of a cylinder of metal (namely a platinum iridium alloy) which is known as the "International Prototype Kilogram." As nearly as possible exact replicas of this were made and are preserved in other countries.

The pound is 0.4535924 times the force of gravity acting upon the international prototype kilogram when located at sea level and a latitude of 45 degrees. The above locality will be referred to as the "standard locality."

If a quantity of some base metal be made in such amount that its weight is 0.4535924 times that of the international prototype kilogram as indicated by a beam scales, this quantity of matter is what is colloquially (improperly in this study) referred to as a "weight of one pound." It is not the material, but the force of gravity between it and the earth, which constitutes "weight." If the material mentioned above be located at an altitude greater than sea level the force of gravity between it and the earth is less than one pound. It cannot be used without correction for the measurement of forces by the beam scales method if great precision is required. For ordinary engineering purposes it can be used since the

gravity forces upon objects near the surface of the earth is less than one-half of one per cent of its value at sea level and 45 degrees latitude.

Another method of measuring force depends upon the elastic property of material by virtue of which it will elongate or contract in proportion to the force applied. The spring balance is the commonest example; others are the steam-pressure gage, the steam-engine indicator, spring dynamometer, etc. Such an instrument may be standardized by applying to it a number of so-called " pound-weights " at *standard* locality, and noting the amount of spring deformation per unit of gravity force. Once standardized it may be used to measure any kind of force, at any locality, without correction, were it not for mechanical imperfections, friction and the like.

77. The unit of mass. It follows from Newton's Second Law (Art. 75), that if an unbalanced force, F, acts upon a body, the motion of the body is accelerated in the direction of the force. The amount of acceleration is directly proportional to the force and inversely proportional to the mass of the body. Expressed mathematically

$$F \text{ varies as } M \times A.$$

This is a general form in which F may be the resultant of a number of forces, of which gravity may be one; and the acceleration, A, is in the direction of the resultant.

It is convenient to choose the " unit of mass " so that the preceding equation may be written

$$F = M \times A.$$

That is, when F equals one pound and A equals one foot per second per second, M will be unity and the proportionality factor will be one. By experiment it has been found that if gravity acts upon a body at the standard locality (air resistance and other disturbing conditions being corrected for) the body will be accelerated downward at the rate of 32.174 feet per second per second. This is true no matter what the mass of the body, since gravity force is increased if the mass is increased, in exact proportion (Art. 8). Using the form, $F = M \times A$, when gravity alone acts,

$$W = M \times 32.174 = M \times 32.2 \text{ (approximately)},$$

whence $$M = W \div 32.174 = W \div 32.2 \text{ (approximately)}.$$

In this equation M equals unity when W equals 32.174 or 32.2 (approximately), and therefore in $F = M \times A$ a force of one pound will produce unit acceleration upon a body weighing 32.174 pounds at standard locality.

The engineer's "unit of mass," from this, is that of a quantity of material weighing 32.174 pounds at the standard locality. For ordinary calculations the number 32.2 may be used.

Various names have been suggested for this unit of mass, such as "slug," "gee-pound," etc., but none has been adopted generally. In this work it will be referred to simply as "unit of mass."

In most works on Mechanics, the symbol "g" is used to indicate the acceleration of gravity at any location. In this book, the symbol g is not used. Instead, its approximate value, 32.2, is written in all equations involving the acceleration of gravity. This is sufficiently accurate for ordinary engineering problems with data pertaining to locations in the United States.

It should be understood, however, for theoretical accuracy in the use of the relation, $F = MA$, that for M should be substituted $W \div g$, in which W is the actual force of gravity (as measured by a spring scales) upon the body whose mass is M, and g is the actual acceleration of gravity at the location of the body.

In the following problems two localities are referred to, p and q. p is the "standard locality" and q is at an elevation where the acceleration of gravity is 32.00 feet per second per second.

PROBLEM 186. A quantity of sugar is weighed at p by a beam scales which indicate 10 lbs. If the same scales are used at q, will the same, more or less sugar be weighed at an indication of 10 lbs.?

PROBLEM 187. Same conditions as Problem 186 except that a spring balance is used.

PROBLEM 188. An engine delivers a turning moment of 10 lb.-ft., as shown by a measured force, F, acting at a 1-ft. moment arm. F is measured at p by a beam scales. If the engine is moved to q and delivers the same turning moment, will the beam scales indicate the same, more or less than F lbs.?

PROBLEM 189. Same conditions as Problem 188 except that a spring balance is used.

PROBLEM 190. A pressure gage indicates 20 lbs. per sq. in. when attached to the bottom of a water tank, the level of the water being L feet above the gage, locality at p. What will the gage indicate at q, under the same hydrostatic head of L feet?

PROBLEM 191. What would be the gravity force in pounds on the international prototype kilogram, if located at q?

There is another unit of mass in the English system, namely, "the pound mass." It may be defined as follows:

One pound mass is 0.4535924 times the mass of the international prototype kilogram.

The force of gravity acting at standard location on a body of this mass,

as defined under Art. 76, is also called a pound. The name "pound" thus does double duty resulting in confusion as to whether a *pound mass* or a *pound force* is referred to. Furthermore, if these units are applied to the relation $F = kMA$, the proportionality factor, k, is no longer unity; for, if F and M are each called one pound, A will still be 32.174 (at standard locality) and the equation is untrue unless $k = 1 \div 32.174$. If it is wished to keep the factor k unity for unit acceleration and a unit mass of one pound, then the force, F, cannot be one pound as previously defined, but a fraction of it, namely, $1 \div 32.174$ of a pound. This fraction, taken as a unit of force, has been given a definite name and subjected to use in scholastic circles. Since it has never been used in any commercial physical laboratory or in engineering literature or practice, its only function being to confuse the student of Mechanics, it will not be named here. It is mentioned only in order to encourage other writers and teachers to join the present tendency to discard it.

78. **The absolute system** is not used in English units, being confined to the metric system. By this method of definition, a unit force is that which gives a unit mass unit acceleration. Thus if the unit of mass is the gram and of acceleration is the centimeter per second per second the unit of force is

$$1 \text{ dyne} = 1 \text{ gram} \times 1 \text{ centimeter per second per second},$$

that is, a dyne is the force necessary to accelerate 1 gram mass with an acceleration of 1 centimeter per second per second.

The English unit of force as defined in Art. 76, together with its related units is called the Gravitational System.

It is to be noted that the Gravitational System uses weight as a fundamental conception, and that the absolute system uses inertia which is a function of weight.

PART IX

KINETICS OF BODIES UNDER UNIFORM RECTILINEAR ACCELERATION

79. Acceleration produced by applied forces can now be calculated from the simple relation $F_r = M \times A$, in which has been substituted for M its approximate value, $W \div 32.2$, and F_r is the resultant of all applied forces including gravity.

PROBLEM 192. Fig. 143 represents a body weighing 64.4 lbs. bearing on a horizontal plane and acted upon by a horizontal resultant force, F, of 10 lbs. What is the acceleration?

Solution. Since there can be no vertical motion, F is the only force to produce acceleration. The acceleration will be in the direction of the vector, A. The mass, $M = 64.4 \div 32.2 = 2$, and the acceleration, $A = F \div M = 10 \div 2 = 5$ ft. per sec. per sec.

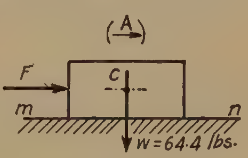


FIG. 143.

PROBLEM 193. An elevator weighing 2500 lbs. is being raised by a cable in which there is a tension of 3000 lbs. What is the acceleration?

Solution. Fig. 144 shows the space diagram and that the resultant force, F_r, is $3000 - 2500 = 500$ lbs. acting upward. The acceleration is therefore upward and is

$$A = F_r \div M = 500 \div (2500 \div 32.2) = 6.44 \text{ ft. per sec. per sec.}$$

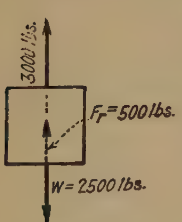


FIG. 144.

PROBLEM 194. If in Problem 193, the elevator is going downward with a tension in its cable of 2035 lbs., what is its acceleration?
Ans. 6 ft. per sec.

PROBLEM 195. The weight of the body shown in Fig. 132 is 100 lbs., $f = 0.3$, after motion is established $f' = 0.15$. With the applied force, F, of 32.6 lbs. which was necessary to start motion (Problem 165) what will be the acceleration and in what time will 100 ft. be covered? *Ans.* 3.73 ft. per sec. per sec., 7.3 sec.

PROBLEM 196. The train of cars shown in Fig. 145 is under a draw-bar pull of 1650 lbs. from the locomotive. If the rolling resistance is 10 lbs. per ton, what will be the acceleration; and the forces on each coupling?
Ans. 0.193 ft. per sec. per sec.; 440, 1100 lbs.

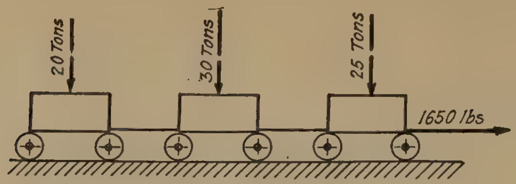


Fig. 145.

80. Theorem. *If a body receives an acceleration causing rectilinear motion* (as defined in Art. 65) and the path of every point of the body is a straight line, *then the resultant force causing acceleration must pass through the center of gravity of the body, and be colinear with the path of the center of gravity.*

In Fig. 146 let the body shown be acted upon by an unbalanced force, F_r, which is the resultant of all applied forces including friction and gravity. By the definition of rectilinear motion every particle of the body must receive the same acceleration. Considering a differential mass, dM, of the body, the force, dF, required to accelerate it is

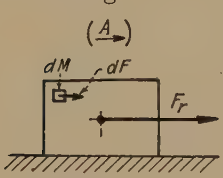


Fig. 146.

$$dF = dM \times A = \frac{dW}{32.2} \times A,$$

that is,
$$dF = \text{a constant} \times dW.$$

Since all such differential forces are proportional to the corresponding differential weights, their summation, F_r, acts in the same way as gravity, that is, through the center of gravity.

81. Kinetic reactions. Consider the case of a body under an *applied* force, F, which does *not* pass through the center of gravity, as in Fig. 147. It is assumed that this body is supported by a horizontal frictionless plane at two contacts, m and n, equidistant from the center of gravity as shown. R and R' are equal and represent the reactions against the weight, W, when the body is at rest.

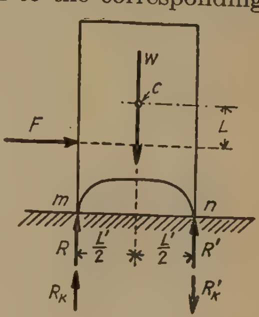


Fig. 147.

Since F does not pass through the center of gravity, c, (Fig. 147) it tends to rotate the body about point c with a moment equal to $F \times L$.

Nevertheless the resulting motion is rectilinear, and not rotation, provided F is not too large. The tendency toward anti-clockwise rotation, FL, must be offset by other forces not yet determined. These forces must be a couple applied at m and n, shown in Fig. 147 by the vectors R_k and R_k', and must produce a clockwise turning moment equal to FL, or

$$FL = (R_k + R_k') \times L' \div 2,$$

Also since there is no vertical motion, the sum of the vertical forces, or components, must equal zero. By examination of Fig. 147,

$$W - (R + R_k) - (R' - R_k'), = 0.$$

Since $$W = R + R',$$

it follows, $$R_k = R_k',$$

and $$FL = R_k L'.$$

If F be increased to such a value that $R_k' = R'$, the reaction at n then equals zero, at the point m equals W, and the body is on the verge of rotation.

It is interesting to note the conformity of this case with the theorem of Art. 80. According to it, the resultant of all the forces must pass through the center of gravity if there is to be rectilinear acceleration. The forces are F, W and the reactions $(R + R_k)$ and $(R' - R_k')$. Since $W = R + R'$, these vertical forces balance and need not be considered. There remain, then, a couple $R_k L'$ and a force, F. According to the theorem of Art. 32, the resultant will be a force, F', parallel and equal to F at a distance sufficient to produce a moment about any point on F equal to the moment of the couple. Since the moment of the couple here considered equals $R_k \times L'$ which in turn equals $F \times L$, the resultant, F', is at a distance L from F and therefore passes through the center of gravity.

82. D'Alembert's Principle. It has been shown that an unbalanced set of forces acting upon a body, producing rectilinear acceleration, has a resultant which passes through the center of gravity of the body. This resultant equals the mass of the body, $W \div 32.2$, times its acceleration, A, and acts in the direction of the acceleration. If on the space diagram for such a problem, a vector be drawn, equal to $WA \div 32.2$, passing through the center of gravity of the body; and parallel and opposite to its acceleration, the space diagram is one of equilibrium, and the corresponding problem in Kinetics may be solved by the conditions of equilibrium of Statics.

D'Alembert and other physicists state that the added force so represented (referred to as the "*reversed effective force*,") is an *imaginary* one. About this there is room for discussion (see Art. 83), but at all events, its use simplifies certain problems in Kinetics.

PROBLEM 197. In Fig. 147, take $A = 6.44$ ft. per sec. per sec., $W = 50$ lbs., $L = 1.5$ ft., $L' = 4$ ft., and the distance of the center of gravity, c, from the supporting plane equal to 4.5 ft. What are the reactions at m and n (*a*) by the principle of Art. 81, and (*b*) by D'Alembert's principle?

Solution. (*a*) The force $F = WA \div 32.2$, since there is no other horizontal force. Hence,

$$F = \frac{50 \times 6.44}{32.2} = 10 \text{ lbs.}$$

The moment of this force (without regarding sign) about the center of gravity is

$$FL = 10 \times 1.5 = 15 \text{ lb.-ft.}$$

This equals the moment of the kinetic couple, $R_k L'$. Therefore,

$$R_k = 15 \div 4 = 3.75 \text{ lbs.}$$

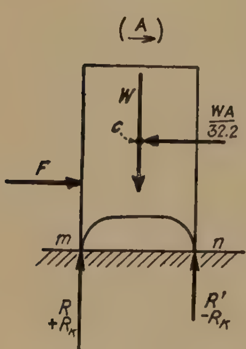


FIG. 148.

The static reaction, R, is one-half the weight, or 25 lbs. At m the total reaction is

$$R + R_k = 25 + 3.75 = 28.75 \text{ lbs.},$$

and at n, the total reaction is

$$R' - R_k' = 25 - 3.75 = 21.25 \text{ lbs.}$$

Solution (*b*). The reversed effective force equals

$$50 \times 6.44 \div 32.2 = 10 \text{ lbs.}$$

The space diagram, Fig. 148, shows this force acting through the center of gravity and parallel to and opposing the acceleration.

Taking moments of all forces about point m,

$$F \times 3 + WL' \div 2 = \frac{WA}{32.2} \times 4.5 + (R' - R_k) \times L'.$$

But $F = 10$ lbs. as under (*a*), and $WA \div 32.2 = 10$ lbs., then

$$10 \times 3 + 50 \times 2 = 10 \times 4.5 + (R' - R_k) \times 4,$$

from which $R' - R_k$, the reaction at n, equals 21.25 lbs., as before. The reaction at m may be found by subtracting this from the weight, 50 lbs., or by taking moments about n.

PROBLEM 198. In Fig. 147, take F equal to 25 lbs. and assume a friction force to be in action at point m equal to 10 lbs., other data (excepting the value of A)

being the same as for Problem 197. What are the normal reactions at points m and n? *Ans.* 23.125, 26.875 lbs.

PROBLEM 199. Using the data of Problem 197 (excepting the value of A) what acceleration will cause the body to tip about the point m? (Note that there is no friction.) *Ans.* 42.9 ft. per sec. per sec.

PROBLEM 200. Same data as Problem 199, except that there is friction, with $f = 0.2$. What is the least value of F which will cause the body to tip around point m? (Note: When body tips the reaction at $n = 0$.) *Ans.* 96.7 lbs.

PROBLEM 201. If F, in Fig. 147, is applied horizontally through point c what are the total reactions at m and n?

PROBLEM 202. If F, in Fig. 147, is applied horizontally *above* point c, what is the effect upon the total reactions at m and n as compared with reactions found in Problem 197?

The effect of friction in problems like the preceding is (a) to increase the force required for a given acceleration, and (b) to slant, from the normal, the reactions at each support by the angle of friction. These reactions will then have two components, R_n and $\overline{F}$ for reaction $(R + R_k)$ and R_n' and $\overline{F}'$ for $(R' - R_k')$. Since the friction at each support equals the coefficient of friction times the normal reaction at that support, the frictional resistance at one support does not equal that at the other.

PROBLEM 203. Using the data of Problem 197 (Fig. 147) except for the value of A, and taking $f' = 0.1$, and $F = 10$ lbs. what are the normal reactions and friction forces at points m and n.

Solution. Fig. 149 shows the space diagram, the reversed effective force indicated by the dotted vector marked $\frac{WA}{32.2}$. The friction forces, $\overline{F}$ and $\overline{F}'$, are known to act as shown, but their individual magnitudes are unknown. But it is known that

$$\overline{F} + \overline{F}' = 0.1\ (R_n + R_n') = 0.1 \times 50 = 5 \text{ lbs.},$$

since the sum of the normal reactions equals the weight.

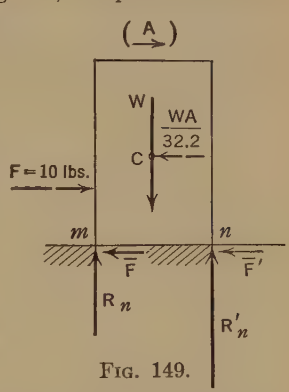


FIG. 149.

The effective force causing acceleration is $F - (\overline{F} + \overline{F}')$ or $10 - 5 = 5$ lbs. The reversed effective force (equal but opposite to) the force causing acceleration, or $WA \div 32.2$ is therefore 5 lbs., which may be marked on the space diagram as shown.

Taking moments about point m (eliminating moment of $\overline{F}$ and $\overline{F}'$),

$$10 \times 3 + 50 \times 2 = 5 \times 4.5 + R_n' \times 4,$$

from which
$$R_n' = 26.9 \text{ lbs.},$$

and $R_n = 50 - 26.9 = 23.1$ lbs.;

also (at point n) $\overline{F}' = 0.1 \times 26.9 = 2.69$ lbs.

and (at point m) $\overline{F} = 0.1 \times 23.1 = 2.31$ lbs.

In this case, it will be observed that the reaction at the right support is greater than the left, the reverse being true in Problem 197 in which there was no friction force. The reason for this in the present problem is that the clockwise moment of the friction force about the center of gravity is greater than the anti-clockwise moment of the applied force, F; the net effect being a tendency towards clockwise rotation, which must be offset by a greater normal reaction on the right than on the left. If F is made equal to 15 lbs., its moment about c ($= 15 \times 1.5 = 22.5$) would equal the opposite moment of $\overline{F}$ ($= 5 \times 4.5 = 22.5$ lb.-ft.); $R_n = R_n'$, and the acceleration would be due to a net force of 10 lbs. ($= F - \overline{F}$). If F is increased to any value greater than 15 lbs., R_n is greater than R'_n by similar reasoning.

Problem 204. An automobile weighs with driver 1800 lbs. The center of gravity is located as shown in Fig. 150. It is brought to rest by a friction force, $\overline{F}$, at the rear wheels with a retardation of 8 ft. per sec. per sec. There is no appreciable friction at the front wheels. What are the normal components of the reactions at the front and rear wheels respectively? *Ans.* 1,068,732 lbs.

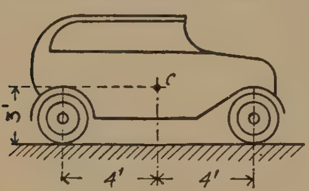


Fig. 150.

Problem 205. What is the maximum possible forward acceleration of the car of Problem 204 if the coefficient of friction between the tires and roadway is 0.667? *Ans.* 14.3 ft. per sec. per sec.

Problem 206. If the coefficient of friction between tire and roadway of the car of Problem 204 is 0.667, what is the shortest distance in which the car could be stopped, from a speed of 20 miles per hour, assuming uniform retardation? *Ans.* 50.1 ft.

Problem 207. If the coefficient of friction between tires and roadway is taken as 0.667 what is the shortest distance in which a car may be stopped, at uniform retardation, from a speed of 20 miles per hour if equipped with four-wheel brakes? No other data are necessary. *Ans.* 20.2 ft.

Problem 208. The manufacturers of a certain car advertise an acceleration of from 5 to 25 miles per hour in 8 seconds. Assuming that the weight of the car and driver is 3000 lbs. and that it has the dimensions of Fig. 150, what is the friction force between tires and roadway and what is the coefficient of friction, if the acceleration is assumed to be uniform?

Ans. 342 lbs., 0.21.

PART X

KINETICS OF BODIES UNDER VARIABLE RECTILINEAR ACCELERATION

83. Inertia Force. It is the custom of engineers to represent as *real*, the reversed effective force which D'Alembert called *imaginary*. Conceived of as a real force, it is called *inertia force* in motion of translation, and *centrifugal force* in uniform rotation. A definition follows:

An inertia force is a reaction from an accelerated body equal and opposite to the force, or the resultant of a number of forces, causing the acceleration of the body. The inertia force passes through the center of gravity of the body and is opposite to the acceleration of the body. It is equal to the mass of the body times its acceleration.

Superficial thinking leads one to believe that an inertia force is a real force. For example, if one applies a muscular force to a body in order to accelerate it, one is conscious of a feeling of resistance, caused by the *inertia* of the body, and therefore apparently of a real reactive force. On the other hand, it may be argued that no acceleration can be accomplished without the existence of a single unbalanced force, and that the only force existing in the cited case is the muscular effort, allowing that gravity is balanced by reactions.

Another illustration is the effect felt by a passenger of a street car who is standing upright, without arm support, when the street car is accelerated forward. He is conscious of an apparent force tending to throw him backward, and, in speaking of it, he would say, " I was thrown backward," implying the action of a force. To refute this, one may say that the body of the passenger was not " thrown," but was merely responding to Newton's first law of motion, restated thus: " A body under a constant velocity in a straight line remains in that condition unless acted upon by an unbalanced resultant force." The passenger was obeying this law; his body was maintaining uniform motion; but the car, upon increasing its velocity, took a different relative position in space from that of the passenger, leaving the latter, as might be said, " in the lurch."

This is plausible, but one should reason a little further. The force of gravity and the force of inertia (if there be such) have two characteristics in common, and different from all other mechanical forces.

(*a*) They are each a direct function of the mass of the body considered, and they each pass through the center of gravity of the body.

(*b*) They are each existent without actual contact between two bodies.

Gravity between the earth and any body near its surface acts without contact in some way not well understood. *Inertia force* acts through the center of gravity without contact of a body during acceleration. But there is no more tangible evidence of the existence of gravity force than there is of inertia force, that is, by the effects they produce. The thing cannot be seen, as can a muscular push or pull, but only felt.[1]

In this book henceforward, the term "inertia force" will be used, with the significance of the definition at the beginning of this article. Its vector may be placed on a space diagram for a problem and the problem will then be one of statics.

84. Reciprocating engines: The Scotch Yoke. Two mechanisms will be studied: the *Scotch Yoke* and the *Crank-Connecting-Rod-Piston* combination. Both these mechanisms have the function of converting variable reciprocating motion into (very nearly) uniform rotary motion, thereby transmitting the translatory effort of the pressure of gas or vapor in a cylinder into rotational effort of a shaft.

Figure 151 shows an engine with a Scotch Yoke, and names its parts.

It is to be remembered, in this analysis, that the crank pin has a constant angular velocity which constrains the reciprocating mechanism to move with variable velocity and acceleration. The piston in its cylinder is under fluid pressure forcing it to the right. Starting from the left-hand extreme position, or head end dead center (abbreviated H.e.D.c.), the reciprocating parts (that is, the piston, piston rod and slotted cross-head) are accelerated to a maximum velocity at mid-stroke. Thence to the right-hand extreme of travel, for crank end dead center (abbreviated C.e.D.c.), the velocity of the reciprocating parts is decreasing, reaching a value of zero at the end of this stroke (called the forward stroke). From the beginning of the return stroke to its end, the same sequence of velocities, in reverse order, takes place as on the forward stroke.

[1] The ideas expressed above are due to Einstein's theory of relativity, in which he gives a very convincing demonstration of the connection between gravity and inertia force, to the effect that the latter may be a real force. This is very clearly and simply expounded in Webster's "General Physics for Colleges," Century Company.

The crank pin, during a double stroke of the piston, is constrained to move vertically in the slotted cross-head, but, since the crank pin is rigidly fastened to the crank and main shaft, the motion of which is rotary, the center of the crank pin traverses the circumference of a circle.

There is a flywheel on the main shaft, the mass of which tends to keep the linear velocity of the crank center constant; that is, the flywheel mass is large enough to resist angular acceleration, except in such small amounts as to be negligible in this analysis; hence the angular velocity of the crank, crank pin, etc., is constant. It will be observed that some of the effort of the working medium in the cylinder is absorbed in accelerating the reciprocating parts, thereby detracting from the force transmitted to the crank. This effort is recovered when the reciprocating parts are retarded.

The engine frame supports the cylinder and the shaft bearings, holding

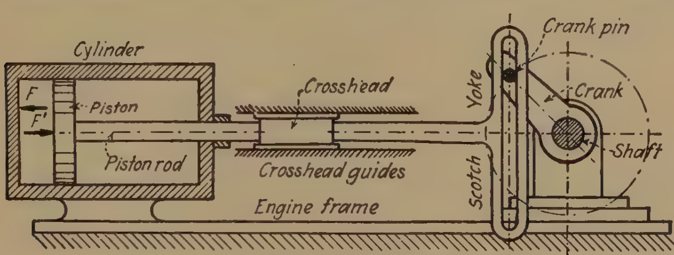


Fig. 151.

them rigidly in their relative positions. In Fig. 151, the forces, F and F', equal each other and are due to the expansive effort of the working medium. F tends to push the cylinder, frame and main bearings to the left. If there were no inertia force of the piston, rod and Scotch Yoke, F' would in equal amount tend to push the crank pin, and hence the shaft, to the right; the forces at the main bearings then would balance. But, since the reciprocating parts are accelerated, the horizontal force transmitted through the piston to the shaft is less than F' by the amount required for the acceleration of the reciprocating parts. There is then an unbalanced pair of horizontal forces between the shaft and main bearings, tending to push the whole engine to the left. The reverse applies upon retardation. Bodily motion of the engine is prevented by its foundation bolts, but, at high rotative speeds, the vibrations set up must be counteracted in a way which will be described later.

The usual reciprocating steam-engine mechanism as shown in Fig. 158 is subject to a similar analysis, the motion being a modified form of that effected by the Scotch Yoke.

A complete analysis of either mechanism must take into account the forces acting on the piston; consisting of fluid pressure of the working medium, inertia forces, and the reactions caused by both, at a number of piston positions.

Concerning inertia forces of the reciprocating parts, it is necessary first to determine the magnitude and direction of the acceleration, A, of the reciprocating parts at any crank angle or at any position of the piston; and, second, from this to find the corresponding inertia force, $M \times A$, the mass, M, of the reciprocating parts being known. It is to be noted that the value of A is not constant.

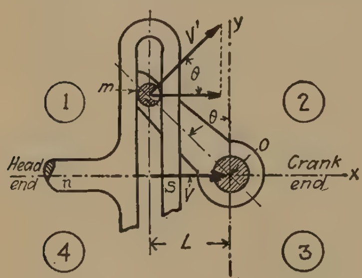


Fig. 152.

85. Accelerations of the Scotch Yoke. Figure 152 is a diagram showing the crank, *mo*, and the slotted cross-head, *ns*, in any position. The displacement of both the cross-head and the crank will be measured from their middle position, *oy*, as shown in Fig. 152, and by the following notation:

Let θ = displacement in radians of crank *from oy*,
L = displacement in feet of the cross-head *from oy*,
r = radius of the crank in feet,
V = linear velocity in feet per second of the point s, which is a point on the slotted cross-head,
V' = linear tangential velocity in feet per second of the point m, the center of the crank pin (constant),
ω = angular velocity of point m, in radians per second,
A = linear acceleration in feet per second per second of point s, and therefore of the reciprocating parts.

The velocity of the point, s, is the horizontal component of that of the point m. Therefore,

$$V = V' \cos \theta,$$

and since

$$V' = r\omega$$

$$V = r\omega \cos \theta .$$

Differentiating this with respect to time:

$$dV \div dT = A = -r\omega \sin \theta d\theta \div dT,$$

and, since $$d\theta \div dT = \omega \text{ and } r \sin \theta = L,$$

$$A = -\omega^2 L.$$

This is the equation of "simple harmonic motion" (abbreviated S. h. m.) in which the constant is $-\omega^2$.

If L is called plus when the cross-head is to the right of the middle position, and minus when to the left; and if the accelerations to the right are called plus and to the left minus (note carefully the locations of the quadrants in Fig. 152), then, since A varies as $-L$

When the crank is in the first quadrant A is plus,
When the crank is in the second quadrant A is minus,
When the crank is in the third quadrant A is minus,
When the crank is in the fourth quadrant A is plus,

and the acceleration always points toward the middle position of the reciprocating parts no matter in which direction they are moving.

Since the acceleration, A, varies as the displacement, L, from the middle position, A is a positive maximum at the H.e.D.c. position and a negative maximum at the C.e.D.c. position. A curve showing the variation is depicted in Fig. 153.

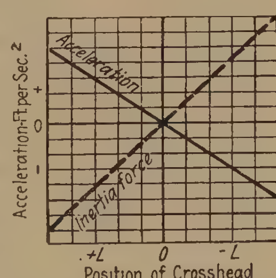


Fig. 153.—Accelerations and Inertia Forces of the Reciprocating Parts of a Scotch Yoke Mechanism.

The physical interpretations of these relations may be explained as follows, if it is remembered that the velocity of the reciprocating parts, undergoing rectilinear motion, is the projection of the constant velocity of the crank pin, undergoing rotary motion; this projection being on the path of a point on the center line of the engine and midway in the slot of the cross-head.

(a) When the crank is at either dead center, the reciprocating parts have a zero velocity, but the greatest change in velocity occurs at these points; hence the acceleration is maximum.

(b) When the crank is vertical, the velocity of reciprocating parts equals that of the crank, and is maximum. During the motion of the crank center from one mathematical point to the left of its middle position,

and one to the right of that position, there is no change in the linear velocity of the reciprocating parts; hence the acceleration is zero in mid-position.

(*c*) When the reciprocating parts move out from their mid-position towards either dead center, their velocity is *decreasing*; when they move in towards the mid-position from either dead center, their velocity is *increasing*. Hence the acceleration in all cases points to the mid-position.

PROBLEM 209. The radius of the crank of a Scotch Yoke mechanism as shown in Fig. 151 is 5 ins. The linear acceleration of the cross-head at H.e.D.c. is 0.5 ft. per sec. per sec. What is the acceleration when the displacement is 2 ins. to the left of mid-position?

Solution. Since the linear acceleration at H.e.D.c. is 0.5 ft. per sec. per sec. $A = 0.5 = -\omega^2(-\frac{5}{12})$ whence $\omega^2 = 1.2$ (the displacement at H.e.D.c. is 5 ins. or $\frac{5}{12}$ ft.) and the linear acceleration, since ω is constant, at a cross-head position 2 ins. to the left, where $L = -\frac{2}{12}$, is

$$A = -1.2(-\tfrac{2}{12}) = 0.2 \text{ ft. per sec. per sec.}$$

When $L = 4$ in. $= \frac{4}{12}$ ft. to the right of mid-position,

$$A = -1.2(\tfrac{4}{12}) = -0.4 \text{ ft. per sec. per sec.}$$

Or:

Since the acceleration of the reciprocating parts varies directly as the displacement from mid-position, A may be calculated by proportion:

$$0.5 : (-5) = A : (-2)$$

whence A, at 2 ins. to the left is

$$0.5 \times 2 \div 5 = 0.2 \text{ ft. per sec. per sec.,}$$

and at 4 ins. to the right,

$$0.5 : (-5) = A : 4,$$

whence $$A = -0.4 \text{ ft. per sec. per sec.}$$

Since A points always toward mid-position, it makes no difference in which quadrant is the crank, and it is only necessary to know on which side of mid-position is the displacement.

PROBLEM 210. In the Scotch Yoke of Problem 209 what is the acceleration of the cross-head when it is 1 in. to the left of mid-position? When the crank is in the second quadrant and the cross-head 3 ins. from mid-position?

Ans. 0.1, −0.3 ft. per sec. per sec.

PROBLEM 211. The radius of the crank of a Scotch Yoke mechanism is 2 ins. and the revolutions per minute are 500. What is the maximum acceleration of the cross-head? Draw a diagram to show the positions of the crank, and the accelerations as vectors, for both dead center positions. *Ans.* ± 450 ft. per sec. per sec.

PROBLEM 212. Using the data given in Problem 211 what is the maximum acceleration when the revolutions per minute of the crank is 1000 instead of 500? when 1500?

Ans. 1800, 4050 ft. per sec. per sec.

PROBLEM 213. The radius of the crank of a Scotch Yoke mechanism is 10 ins. and the crank makes 120 r.p.m. Draw four positions of the crank, each inclined 45° from the horizontal, and calculate the corresponding accelerations of the reciprocating parts and represent these by vectors on the diagram.

Ans. $A = \pm 92$ ft. per sec. per sec.

86. Inertia forces on the Scotch Yoke. Let W represent the weight in pounds of the reciprocating parts. Their mass is then $W \div 32.2$, and the inertia force is

$$F_i = -\omega^2 LW \div 32.2.$$

When the mechanism is on either dead center, $L = r$, if r be taken as the crank length in feet. Substituting this in the above equation and remembering that the value of ω is $2 \times 3.14 \times N \div 60$ and combining constants, we have

$$F_i \text{ at dead center} = \pm 0.00034\, WrN^2.$$

Since the inertia force is always opposite to the acceleration, and the acceleration is always pointing *towards* mid-position, the directions of the inertia force in any position of the reciprocating parts is easily assigned; it is always *away* from the mid-position.

According to Art. 85, accelerations of the Scotch Yoke and attached parts to the *left* of the mid-position, *oy* (Fig. 152) are plus, when to the right, minus. Therefore (see Fig. 152):

In the first and fourth quadrant, A is plus, and F_i is minus,
In the second and third quadrant, A is minus, and F_i is plus.

The physical aspect of these conclusions is as follows: When the piston starts its forward stroke, the working medium in the cylinder exerts a force, F, on the piston, to the right (plus) and the inertia force is to the left (minus). The algebraic sum of these forces, $F - F_i$, is transmitted to the crank pin which is under less turning effort, when the crank is in the first quadrant, than would exist if there were no inertia force. The opposite is true when the crank passes the vertical; the active force in the cylinder, which still acts to the right, is *assisted* by the inertia force which now acts to the right, increasing the net force transmitted to the crank, until C.e.D.c. is reached.

The variation of the inertia force at intermediate positions of the crank follows the same law as does the acceleration, and may be represented graphically by a curve of the same form as Fig. 153, and is shown by the dotted line.

PROBLEM 214. Find the maximum inertia force of the reciprocating parts of a Scotch Yoke mechanism, assuming S.h.m. if the weight of the reciprocating parts is 10 lbs., radius of crank is 2 ins., and N is 1000. Show on a space diagram.

Soultion. The inertia force has the same maximum value at the two centers and equal (without regard to sign),

$$F_i = .00034\ WrN^2$$
$$= .00034 \times 10 \times \tfrac{2}{12} \times 1000^2 = 567 \text{ lbs.}$$

Fig. 154 is the space diagrams, with F_i shown at the two dead center positions, each equal to 567 lbs. but opposite in direction.

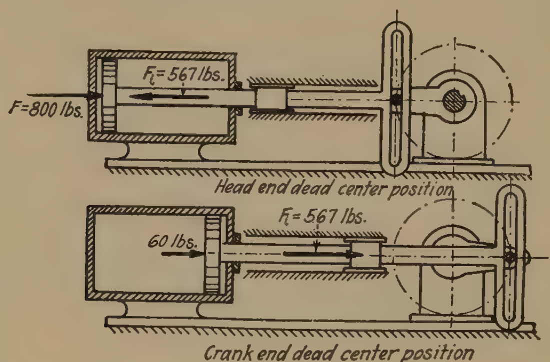


FIG. 154.

PROBLEM 215. With the same data as for the preceding problem, consider the effect of a fluid pressure on the head end of the piston, equal to 800 lbs. at H.e.D.c. and equal to 60 lbs. on the C.e.D.c. What is the force delivered to the crank pin? What are the reactions at the bearings?

Solution. The vector F in Fig. 154 indicates the fluid pressure, and acts to the right. Therefore, at H.e.D.c. the force transmitted to the crank pin is

$$800 - 567 = 233 \text{ lbs.}$$

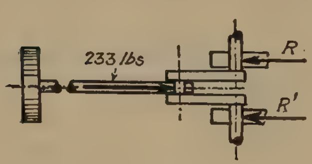


FIG. 155.

and at the C.e.D.c.

$$60 + 567 = 627 \text{ lbs.,}$$

Fig. 155 is a plan view of the engine showing its crank and bearings at H.e.D.c. The force transmitted to the crank pin is 233 lbs. and is shown by a vector so marked. The force is transmitted through the crank and tends to push the bearings to the right. The two reactions, R and R', *from* the bearings *to* the crank shaft together equal 233 lbs. or else they would not be in equilibrium with

the crank shaft. If the bearings are equidistant from the center line of the cylinder, the reaction,

$$R = R' = 233 \div 2 = 116.5 \text{ lbs.}$$

At C.e.D.c. the reactions are, similarly, $627 \div 2 = 313.5$ lbs.

Between these two positions, the reactions vary according to the algebraic sum, at any crank position, of the fluid pressure and the inertia force.

PROBLEM 216. Find the inertia forces and the reactions at bearings, at dead center positions of the crank, for a Scotch Yoke mechanism, if $W = 12$ lbs., $r = 3$ in. and $N = 500$. Draw space diagrams as in Figs. 154 and 155, taking the fluid pressure at 850 lbs. on the H.e.D.c. and 75 lbs. on the C.e.D.c.

Ans. $F_i = 255$ at each end; $R = 297.5$ lbs. on each bearing at H.e.D.c. and $R = 165$ lbs. at C.e.D.c.

PROBLEM 217. Using the data of Problem 214, what is the inertia force when the crank has moved 45° from the H.e.D.c.? When the crank has moved 90° from H.e.D.c.? What are the forces transmitted to the crank pin if the fluid pressure at 45° is 475 lbs. and at 90° is 325 lbs.? *Ans.* $F_i = 400$, 0 lbs.

PROBLEM 218. Given two parallel cylinders with cranks 180° apart as shown in Fig. 156 in which it is assumed that each piston moves under S.h.m. The weight

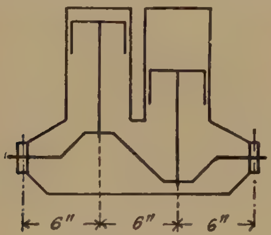


FIG. 156.

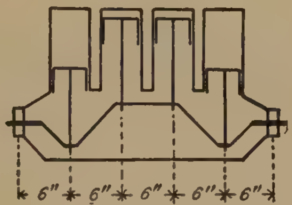


FIG. 157.

of the reciprocating parts in each cylinder is 10 lbs., radius of crank 2 ins. and the revolutions per minute is 1500. The fluid pressure at H.e.D.c. of each piston is 800 lbs., and C.e.D.c. is 60 lbs. Draw the space diagram, with calculated bearing reactions for the position shown by Fig. 156, and for a second position of the crank 180° from the first.

PROBLEM 219. Same data and questions as for Problem 218, except that there are four cylinders as shown by Fig. 157.

In solving the preceding problems, the form

$$F_i = \pm .00034\, WrN^2$$

may be used for convenience, but the fundamental relation $A = -\omega^2 L$, should not be forgotten, since the above value of F_i is deduced from this equation for acceleration.

87. Vibrations of Engines. In combinations of cylinders like those of Fig. 157, the internal forces would be balanced at the bearings were it not for inertia. The fluid pressure in any cylinder would then act

upwards on the cylinder head and tend to push it and the cylinder away from the engine shaft at the bearing, making a reaction from the bottom side of the bearing (to which the cylinder is attached) *to the shaft.* An equal force is transmitted by the fluid pressure *downward* on the piston, through the crank pin and shaft *to the bearings.* As far as fluid pressure is concerned, it is thus seen that the reaction on the shaft from the upward force on the cylinder head exactly balances the fluid pressure on the piston transmitted to the shaft bearings. The inertia forces disturb this balance by making an upward excess from the bearings to the shaft at H.e.D.c. and a downward deficiency at C.e.D.c. These forces tend to pull the engine as a whole away from its foundation bolts, at the one position and downward upon the engine frame in the other. With high-speed engines, vibrations result which can be nullified or reduced by one or more of the following methods:

(*a*) The balancing of inertia forces at one cylinder by those at one or more other cylinders.

(*b*) By reducing the weight of the reciprocating parts in all cylinders.

(*c*) By attaching "counter-balance weights," to the shaft opposite each piston. (See Art. 96.)

Methods (*a*) and (*c*) would be quite effective if the reciprocating parts were under simple harmonic motion. In the actual high-speed engine, as used for automotive purposes, this is not the case, since the effect of the connecting rod is to modify the motion from S.h.m. and to cause inertia forces in the plane of the connecting-rod oscillation. Only the inertia forces parallel to the cylinder center line will be considered in the following treatment.

88. Acceleration of the reciprocating parts of the conventional engine.

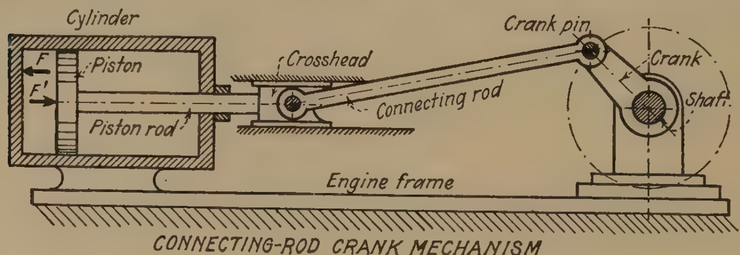


FIG. 158.

The Scotch Yoke has mechanical defects which are obviated by the mechanism shown in Fig. 158. This is repeated diagrammatically by Fig. 159,

the line mn representing the connecting rod in the same position as in Fig. 158. The corresponding position of the cross-head is at n. When on H.e.D.c. the cross-head is at p, and when on C.e.D.c. it is at q. The

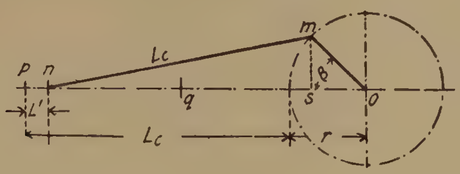


FIG. 159.

reciprocating parts as a whole are under the same displacement and acceleration as the point, n, on the cross-head.

Instead of using the mid-position as a datum for measuring displacements, as was done in the case of the Scotch Yoke, *the H.e.D.c. position will be used.*

Let L' = the distance, in feet, moved by the cross-head from H.e.D.c. to some position, n,

β = the corresponding angular displacement, in radians, of the crank from H.e.D.c.,

L_c = length of the connecting rod, in feet,

r = length of crank, in feet,

$k = r \div L_c = 0.2$ to 0.25 in the usual design,

A = acceleration of the reciprocating parts when crank is at the angle β.

From Fig. 159: $L' = L_c + r - ns - so$ and, by trigonometry,

$$L' = L_c + r - \sqrt{L_c^2 - r^2 \sin^2 \beta} - r \cos \beta.$$

The term under the radical may be expanded by the Binomial Theorem. Omitting as negligible quantities terms with exponents greater than 2, and introducing k by its equality to $r \div L_c$, the radical becomes

$$L_c(1 - 0.5\, k^2 \sin^2\beta) \text{ very nearly.}$$

Substituting in the last equation for L' and simplifying,

$$L' = r + 0.5\, L_c\, k^2 \sin^2 \beta - r \cos \beta.$$

Differentiate with respect to time and insert for $\sin^2 \beta$ its value

$$(1 - \cos 2\,\beta) \div 2.$$

Differentiate the resulting form with respect to time, to get the acceleration, with the following result:

$$A = r\omega^2 (\cos \beta + k \cos 2\beta),$$

ω being the angular velocity of the crank, or $d\beta \div dT$, in radians per second.

When $\beta = 0°$ (H.e.D.c.) $\quad A = r\omega^2(1 + k)$
$\beta = 90°$ $\quad A = r\omega^2(-k) = -r\omega^2(k)$
$\beta = 180°$ (C.e.D.c.) $\quad A = r\omega^2(-1 + k) = -r\omega^2(1 - k)$
$\beta = 270°$ $\quad A = r\omega^2(-k) = -r\omega^2(k)$

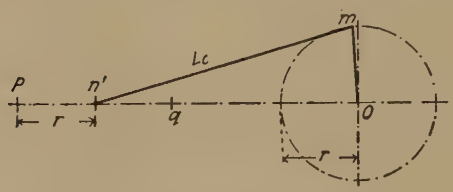


Fig. 160.

It should be noted that when $\beta = 90°$ or $270°$ the cross-head is beyond its middle position as shown by Fig. 160 in which the distance $pn' = r$ and is less than L' for $\beta = 90°$.

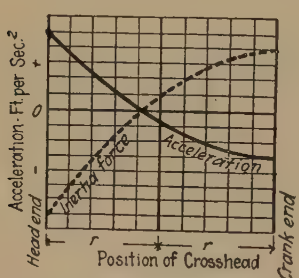


Fig. 161.—Accelerations and Inertia Forces of the Reciprocating Parts of a Conventional Engine Mechanism.

Figure 161 is a curve showing the variation of acceleration with piston position. The difference between values of A for this mechanism and the values of A for a Scotch Yoke is indicated by the parentheses in the equations just written for A.

For example, if $k = 0.2$, the acceleration at H.e.D.c. of the connecting-rod-crank combination is one-fifth greater than for S.h.m., and at C.e.D.c. it is one-fifth less. When the crank is in mid-position (the cross-head being beyond its middle position) A is one-fifth the value of A for S.h.m. at dead center, instead of zero as for S.h.m.

Problem 220. The radius of the crank of a conventional engine is 2 ins. and the revolutions per minute are 1500. The length of the connecting rod is five times the crank radius. What is the acceleration at H.e.D.c.? At C.e.D.c.? When the crank is at 90°? When at 270°?

Ans. 4860, −2430, −810, −810 ft. per sec. per sec.

PROBLEM 221. Same data as in Problem 220 excepting that the revolutions per minute is 500. What are the accelerations at H.e.D.c., C.e.D.c. and the two mid-positions of the crank? *Ans.* 540, −270, −90, −90 lbs.

89. Inertia forces of the reciprocating parts of the conventional engine. As for the Scotch Yoke, the acceleration at any position of the crank is multiplied by the mass of the reciprocating parts (for approximate results, part of the mass of the connecting rod is considered as reciprocating and part as rotating) to obtain the inertia force at this position. A curve of these forces has the same form as the curve of the acceleration (Fig. 161), but in the opposite direction as shown by the dotted curve.

If W is the weight of the reciprocating parts in pounds, including part of the connecting rod, the inertia forces, F_i, are:

$$\begin{aligned}
&\text{at H.e.D.c.} = 0.00034\, WrN^2(1 + k),\\
&\text{at C.e.D.c.} = -\,0.00034\, WrN^2(1 - k),\\
&\text{and at the crank mid-positions} = -\,0.00034\, WrN^2(k),
\end{aligned}$$

which differ from the corresponding values for S.h.m. (Art. 86) only by the value of the factor containing k, in the parentheses.

This is an approximation, although a close one, since in the derivation of the acceleration, A, Art. 88, certain terms resulting from an expansion by the binomial theorem were neglected. It is to be noted that this analysis does not consider the vertical accelerations of the connecting-rod mass, which may be treated independently.

PROBLEM 222. In a conventional engine having a single cylinder the center line of which is equidistant from two bearings the radius of the crank is 2 ins., revolutions per minute are 1500, $W = 10$ lbs., and $k = 0.25$. What are the inertia forces at both H.e.D.c. and C.e.D.c.? Show on a space diagram; what are the reactions at the bearings due to the inertia forces? *Ans.* $F_i = 1595, -957$ lbs.

PROBLEM 223. Same data as in Problem 222 except that there are two cylinders arranged with cranks 180° apart as shown in Fig. 156. Show on space diagrams the reactions on the bearings at the two dead center positions.

PROBLEM 224. Same as for Problem 222 excepting for a four-cylinder arrangement as shown in Fig. 157.

PART XI

UNIFORM ROTATION OF BODIES

90. Uniform rotation of Bodies. Consider a body moving with uniform velocity in the arc of a circle. According to the first law of motion, Art. 75, a body moving at constant velocity, *if not acted upon by an unbalanced force,* remains in its state of uniform motion in a straight line. If, then, the path of the body is that of an arc there must be an unbalanced force constraining the body to deviate from its natural rectilinear motion. For example, Fig. 162 represents a metal ball moving in a circular groove with a constant velocity, V. To constrain the motion from the straight line indicated by the vector, V, in the figure, there must be a force, F_c, at the outside edge of the groove, and this force must be perpendicular to the arc of the groove, Art. 23, and therefore radial.

FIG. 162.

The radial force, F_c, pointing toward the center of the circular path is called the "centripetal force."

91. Radial Acceleration. In the foregoing instance, since there is an unbalanced force acting upon the body, there must be an acceleration toward the center of its path, and, by reasoning similar to that employed in Art. 80, the force causing acceleration must pass through the center of gravity of the body, in the line of the acceleration.

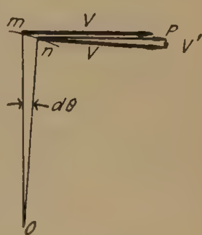


FIG. 163.

In Fig. 163, let m and n be two consecutive positions of the center of gravity of the body rotating about the point, o, a differential distance, dL, apart.

Let A' = radial acceleration of the body in feet per second per second,
dL = length of the path, mn, in feet,
$d\theta$ = corresponding displacement in radians,
ω = uniform angular velocity in radians per second,

V = linear velocity of the center of gravity of the body in feet per second,
r = radius, in feet, of the path of the center of gravity,
dT = time, in seconds, required for the motion from m to n.

Since the body has a constant angular velocity the linear velocity at the point n is equal to that at the point m and is represented in each case by a vector, V, shown with heavy lines, Fig. 163, each tangent to the arc mn. If a vector is now drawn from n, equal and parallel to the vector representing the linear velocity at m, shown by line np, Fig. 163, it is seen that the vector V', closing the vector triangle, is the necessary change in velocity to make the body follow the circular path, mn. Since the angle $d\theta$ is a differential angle, the velocity represented by the vector V' is radial.

Since the time required for this change is dT,

$$A' = V' \div dT.$$

From the figure, $V' = Vd\theta$, and $d\theta = dL \div r$. Combining:

$$A' = VdL \div rdT,$$

but
$$dL \div dT = V,$$

whence
$$A' = V^2 \div r.$$

Substituting for V its value ωr (Art. 72) gives another form,

$$A' = \omega^2 r.$$

92. Centripetal force is the force, F_c, shown by the vector so marked on Fig. 162. Multiplying the two forms given above for the radial acceleration A' by the mass of the rotating body gives two forms for the centripetal force; namely,

$$F_c = \frac{WV^2}{32.2\,r},$$

and

$$F_c = \frac{W}{32.2}\,\omega^2 r,$$

or, in engineering units, after inserting for ω its value, $2 \times 3.14 \times N \div 60$,

$$F_c = 0.00034\ WrN^2.$$

The last two forms show that the centripetal force varies directly as the weight and the radius of the mass center path, at a given rotative speed.

If the mass of a rotating body could be concentrated to that of a particle located at its center of gravity, then the centripetal force would be the same as under the actual distribution of mass.

PROBLEM 225. An eye-rod with the dimensions shown in Fig. 164, weighs 10 lbs. and has its center of gravity 2.5 ft. from the center of rotation. It is revolved at a speed of 500 r.p.m. about a shaft passing through the eye. What is the magnitude of the centripetal force, at what point does it act and what is its line of action?

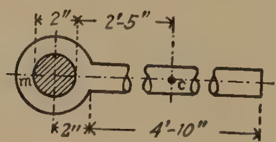


FIG. 164.

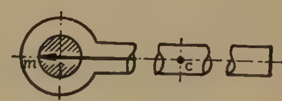


FIG. 165.

Solution. Since the center of gravity is 2.5 ft. from the center of rotation, the centripetal force is

$$0.00034 \times 10 \times 2.5 \times (500)^2 = 2125 \text{ lbs.}$$

Its point of application is the point, m, at which the eye bears on the shaft, and the line of action passes through the center of gravity, c, as shown by Fig. 165. The force is *from* the shaft and *on* the eye-rod.

PROBLEM 226. If the linear velocity of a point on the end of the eye-rod of Problem 225 is 10,000 ft. per min., what is the force *from* the shaft and *on* the eye?

Ans. 858 lbs.

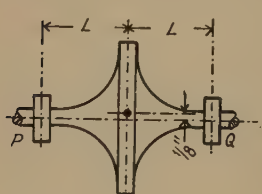


FIG 166.

PROBLEM 227. Fig. 166 shows a turbine rotor which is mounted on a shaft revolving on two bearings, P and Q. The center of gravity of the rotor is $\frac{1}{8}$ in. away from the shaft axis and midway between P and Q. The rotor weighs 1000 lbs. and revolves at 1800 r.p.m. Find the reaction from each bearing on the shaft, first when the center of gravity is above the shaft; and second, 180° from this position.

Ans. 6225 lbs. down, 5225 lbs. up.

93. Centrifugal force. If, in Fig. 165, it is considered that there is a reaction from the eye-rod to the vertical shaft, balancing the centripetal force, this reaction is called the centrifugal force. Since this force is equal to $M \times A$ and is opposite in direction to A, and passes through the center of gravity, it is an inertia force. In this book the centrifugal force will be symbolized by F_c'.

It will be seen that, if the term "centrifugal force" is used as defined in this article, problems dealing with bodies rotating uniformly about an axis may be treated as if in a static state, that is, all the conditions of equilibrium hold. It is a little more evident that centrifugal force is a "real force" than is the so-called "reversed effective force" (Art. 82).

94. The engine governor is a device depending upon centrifugal force for its action. Figure 167 illustrates the mechanism in its simplest form, consisting of two metal balls which are revolved about a vertical spindle, oo', at a rate depending on the engine speed. P, P are rods supporting the balls. Q, Q are links attached to and terminating at a sleeve S. When the engine speed increases slightly the speed of the balls also increases, therefore they rise to a higher plane of revolution and draw up the sleeve, S. To this sleeve is attached a mechanism which controls the amount of working medium, in the case of a steam engine the amount of steam admitted to the engine cylinder. Raising the sleeve is thus followed by a reduced effort in the cylinder and this maintains an approximately constant engine speed.

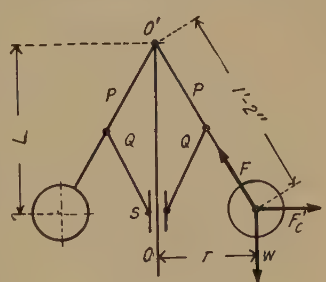


FIG. 167.

Considering one ball only, Fig. 167, it is seen that, when the engine speed is constant there are only three forces (neglecting friction and governor load); namely, F, the tension in the supporting rod, W, the weight, and F_c', the centrifugal force. F_c' passes through the center of gravity of the ball and lies in the plane containing the path of the center of gravity. It may now be considered that the ball is *in equilibrium* under the action of the forces F, W and F_c'. Taking moments about the point o',

$$Wr = F_c' \times L = (W\omega^2 r \div 32.2) \times L,$$

from which

$$L = 32.2 \div \omega^2,$$

or the distance from the mass center to the horizontal plane through o' is inversely proportional to the square of the rotative speed.

PROBLEM 228. Fig. 167. If L is 10 ins., what is the revolutions per minute? If W for one ball is 20 lbs. what is F_c'? Disregard the weight of P.
Ans. 60 r.p.m., 20 lbs.

PROBLEM 229. Fig. 167. The weight of one ball is 20 lbs. Disregarding the weight of P, what is the value of F_c' at 100 r.p.m.?
Ans. 76.5 lbs.

Problem 230. What distance will the balls of the governor of Problem 229 rise when the speed is increased from 100 to 102 r.p.m.? *Ans.* 0.102 in.

Problem 231. It is shown in this article that the distance the governor balls rise is independent of the weight of the balls. Why, then, are large engines supplied with much heavier balls than are smaller engines?

95. The superelevation of the outer rail of a curved railroad track is a problem very similar to that of the engine governor. Figure 168 represents a car travelling with uniform velocity, V, on a track curved to a radius, r. It is required to incline the tracks from the horizontal sufficiently that the reactions from the rails will be perpendicular to mn, Fig. 168, and prevent flange pressure at the outer rail, which would be inevitable on a

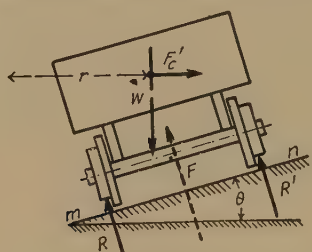


Fig. 168.

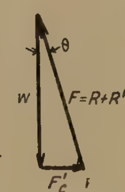


Fig. 169.

horizontal railroad. If the reactions, R and R', are perpendicular to mn, their effect is the same as the force F (shown dotted in Fig. 168) acting through the center of gravity of the car. The car is then in equilibrium under the action of F, W and the centrifugal force, F_c', equal to $WV^2 \div 32.2r$. Drawing the force triangle as in Fig. 169, gives the inclination, θ of the roadway, and

$$\tan \theta = V^2 \div 32.2r.$$

Note that the centrifugal force, when represented by a vector on the space diagram, lies in the plane of the mass center path.

The superelevation of the outer rail equals the distance between the rails, center to center, called the "gage," times the sine of the angle θ, that is, it is the vertical distance of any point on the outer rail above a corresponding point on the inner rail. (See Fig. 168.) For small angles, $\sin \theta = \tan \theta$, very nearly.

Problem 232. Fig. 168 shows a track of 200-ft. radius and a gage of 4 ft. 10 ins. This track is to have a superelevation of the outer rail for a train speed of 25 miles per hour (abbreviated, m.p.h.) What is the superelevation in inches? ($\sin \theta$ does not equal $\tan \theta$). *Ans.* 11.87 ins.

PROBLEM 233. If the track of Problem 232 is built for a train speed of 25 m.p.h. and a car weighing 30 tons passes over it at a speed of 35 m.p.h., what will be the total reactive force from the rails to the wheel flanges? (Solve by components parallel to *mn*, Fig. 168.) *Ans.* 11,520 lbs.

PROBLEM 234. At what speed in feet per second would the car of Problem 233 overturn around the outer rail? The center of gravity of the car is 5 ft. above the tracks. *Ans.* 69.8 ft. per sec.

PROBLEM 235. Fig. 170 represents a flat car travelling at 20 m.p.h. on a horizontal roadway, curved to a radius of 250 ft. A box weighing 500 lbs. rests on the car. What is the magnitude of the friction force holding the box against the centrifugal force? Show on space diagrams, plan view and elevation, and determine the reaction from the floor of the car to the box. *Ans.* 53.3, 502.2 lbs.

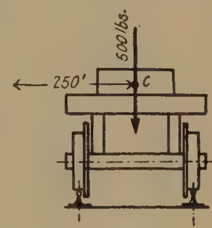


FIG. 170.

PROBLEM 236. If the coefficient of static friction is 0.2 between the box and the floor in Problem 235, at what car speed will the box slide? *Ans.* 40.6 ft. per sec.

PROBLEM 237. Fig. 171 shows a vertical section of a bowl used for motorcycles, the upper part of which has vertical walls. The center of gravity of cycle and driver is 2 ft. from the track. If the coefficient of friction between tires and roadway is 0.5, what linear speed is necessary to maintain the position shown? Draw the space diagram showing the weight, normal reaction, centrifugal force and friction at the wheels. Show that equilibrium is maintained through the balancing of two couples. (Note that since the force of gravity, centrifugal force and friction are all functions of the weight, the required velocity applies to any weight of cycle and driver.) *Ans.* 38.8 ft. per sec.

FIG. 171.

PROBLEM 238. In Problem 237, Fig. 171, can the cycle and rider assume a horizontal position? Why?

96. Balancing reciprocating parts. In Art. 86 the magnitude of the inertia force of the reciprocating parts of the Scotch Yoke at dead center positions was deduced and stated in engineering units to be

$$F_i = \pm\, 0.00034\, WrN^2,$$

r being the radius of the crank expressed in feet.

This is an identical expression to those for centripetal and centrifugal forces as given in Arts. 92 and 93, for a body weighing W pounds, whose mass-center is moving in a circle of r feet in radius.

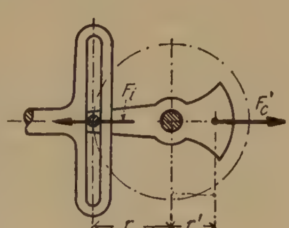


FIG. 172.

Figure 172 represents the slotted cross-head and crank of a Scotch Yoke mechanism. The crank web has an extension on the end opposite the crank pin, the additional metal weighing W' pounds with its center

of gravity r' feet from the shaft center, and diametrically opposite to the crank-pin center.

If W' and r' are proportioned so that $W'r' = Wr$, in which W is the weight of the reciprocating parts and r is the radius of the crank, Fig. 172, then, on either dead center,

$$0.00034\ W'r'N^2 = 0.00034\ WrN^2,$$

or

$$F_c' = F_i.$$

That is, *in the dead center positions,* centrifugal force of the added metal is just sufficient to balance the inertia force of the reciprocating parts.

The centrifugal force, F'_c, is constant in all positions of the crank, but its direction varies with the crank position. The inertia force of the reciprocating parts varies directly with their displacement, L, from mid-position and is always horizontal.

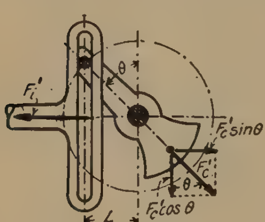


FIG. 173.

Figure 173 represents the mechanism with a crank displacement of θ from mid-position. The centrifugal force is indicated by F_c' and its horizontal component is $F_c' \sin\ \theta$. In the crank position shown the inertia force is indicated by F_i', and as F_i = inertia force at dead center,

$$F_i' = F_i \times L \div r.$$

Since $F_i = F_c'$, and $L \div r = \sin\ \theta$, the right side of the equation becomes $F_c' \sin\ \theta$, whence,

$$F_i' = F_c' \sin\ \theta.$$

That is, the inertia force of the reciprocating parts just balances the horizontal component of the centrifugal force in any position of the mechanism.

Note that the vertical component of the centrifugal force in any position of the crank is unbalanced.

PROBLEM 239. Fig. 173. Weight of reciprocating parts = 10 lbs., radius of crank = 2 ins., r.p.m. = 1000. What weight of material should be added as in Fig. 172 at a center of gravity radius of 1 in.? What unbalanced forces are there when $\theta = 45°$ (Fig. 173) and when $\theta = 0°$? *Ans.* 20 lbs. added.

With the connecting-rod-crank combination, the inertia forces at dead center cannot be exactly balanced by the centrifugal force on an added

mass, because the former is greater on the H.e.D.c. than on the C.e.D.c. as demonstrated in Art. 89. With single-cylinder horizontal engines a compromise may be effected by making

$$W'r' = \left(\tfrac{2}{3}\right) Wr.$$

PROBLEM 240. Radius of crank = 2 ins., r.p.m. = 1500, $k = 0.25$, $W = 10$ lbs. Engine arrangement as in Fig. 158. If the radius of a balancing mass equals 1 in., and $W' \times 1$ in. $= (\frac{2}{3})Wr$, what are the unbalanced centrifugal and inertia forces combined, in a horizontal direction, at $\beta = 0°$, $90°$, $180°$, and $270°$?

PART XII

NON-UNIFORM ROTATION OF BODIES—MOMENT OF INERTIA AND RADIUS OF GYRATION

97. Centripetal and centrifugal forces may be expressed in exactly the same form as for uniform motion; the only difference being that they are not constant since the angular velocity, ω, varies. The values obtained are instantaneous ones, applying to some instantaneous value of angular velocity, or of linear velocity, of the mass-center.

98. Moment of tangential inertia forces. Consider a rotating body, increasing in angular velocity, under a resultant force, F_r, Fig. 174, applied at a distance, L feet, from the center of rotation. The angular acceleration of the body will be the greater, the further away F_r is from the center, since the turning moment, $F_r \times L = \overline{M}$, is then greater. In considering relative acceleration and inertia effects it is therefore necessary to take into account, not only the applied forces or their resultant, but the moments of such forces about the center of rotation.

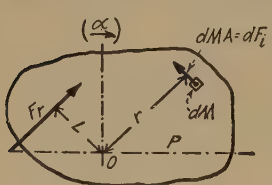


Fig. 174.

Figure 174 represents a body P, rotating about a center o. A particle of mass dM is situated r feet from the center, o, and is under a linear acceleration in the direction of its path equal to A feet per second per second. The inertia force of the particle is shown opposite to the acceleration, and equals

$$dF_i = dM \times A.$$

Multiplying each side of the equation by the radius, r, of the path of dM gives the moment of the inertia force about o

$$r \times dF_i = r \times dM \times A.$$

In Art. 73, it is shown that the relation between the linear accelera-

tion, A, and the angular acceleration, α, in radians per second per second is

$$A = r \times \alpha.$$

Substituting this value of A in the preceding equation, the moment of the inertia force of the particle is

$$\begin{aligned} r \times dF_i &= r \times dM \times r \times \alpha \\ &= r^2 \times dM \times \alpha. \end{aligned}$$

The sum of the moments of the inertia forces of all such particles is

$$\alpha \times \int r^2 \times dM,$$

and this must equal the moment of the resultant of the applied force,

$$F_r \times L = \overline{M}.$$

The quantity $\int r^2 dM$ is called the polar *moment of inertia* of the body and will be referred to in this book as I_z. Then

$$\overline{M} = I_z \times \alpha.$$

The following problems refer to Fig. 175 which represents three small masses, approximating differentials, m, n, and p, fixed relatively by a rigid, weightless wire which revolves with m, n and p about the stationary point, o. Unless otherwise stated, the combination is under constant angular acceleration of 100 radians per sec. per sec. The weights of m, n and p and their distances from o, are 0.161 lb. each and 1, 2 and 3 ft. respectively.

FIG. 175.

PROBLEM 241. What is the linear acceleration in feet per second per second of the particle m? of n? of p? *Ans.* 100, 200, 300 ft. per sec.²

PROBLEM 242. What is the inertia force of m? of n? of p? *Ans.* 0.5, 1.0, 1.5 lbs.

PROBLEM 243. What is the moment about o of the inertia force of m? of n? of p? *Ans.* 0.5, 2, 4.5 lb.-ft.

PROBLEM 244. What moment (produced by a force applied at any distance L, from o to the wire) would balance these inertia forces? *Ans.* 7 lb.-ft.

PROBLEM 245. (a) If the weights of m, n and p are 0.322 lb. each, instead of 0.161 lb., what moment would produce the given angular acceleration? (b) If the weights of m, n and p are 0.161 lb. each, but their distances from o are 2, 4 and 6 ft. respectively, what moment is required for the given angular acceleration? *Ans.* (a) 14 lb.-ft., (b) 28 lb.-ft.

PROBLEM 246. Assume the three particles, m, n and p, weighing 0.161 lb. each, or 0.483 lb. total, to be concentrated at a distance, r_0, from o, instead of being at

the distances of 1, 2 and 3 ft. respectively. Their combined mass is 0.483 ÷ 32.2 = 0.015. The force necessary to give them a linear acceleration at radius r_g corresponding to the same angular acceleration (100 radians per sec. per sec.) is 0.015 × $(r_g \times 100) = 1.5r_g$. The moment of this force about o is $1.5\ r_g^2$. Find r_g.

Ans. $r_g = 2.16$ ft.

PROBLEM 247. Taking the weights of the particles m, n and p as 0.322 lb. each instead of 0.161 lb., at what distance, r_g, from o could they be concentrated and have the same angular acceleration under the same turning moment?

Ans. $r_g = 2.16$ ft.

The student should study the answers of Problems 241–247 in order to understand the physical significance of the general relations involved.

99. Moments of inertia. In rectilinear motion the resistance to acceleration is proportional to the acceleration and to the mass.

In rotational motion, as demonstrated in Art. 98, the resistance to acceleration is proportional to the magnitude of the angular acceleration and *the moment of inertia.*

Problem 243 is an illustration of the fact that the turning moment required to accelerate a mass particle in rotation varies as the square of the distance from the center of rotation. This is further illustrated by Problems 244 and 245 (b).

A notion of the physical meaning of moment of inertia may be had from the following definition:

The moment of inertia of a body rotating about an axis is a measure of the resistance of the body to change in rotative speed by virtue of its inertia, and equals the sum of the moments of the inertia forces of its particles, about the axis of rotation.

The following is a purely mathematical definition:

The moment of inertia of a body rotating about an axis is the sum of the products of the mass of each particle of the body by the square of its distance from the axis of rotation.

The units of this quantity are units of mass × feet^2, or units of mass × inches^2, and it will be referred to in this book by I, with suitable subscripts.

100. Radius of gyration. Problems 246 and 247 illustrate the fact that a summation of mass particles, rotating under the same angular acceleration, could be considered as being concentrated at a certain radius, and that a single force applied to this concentrated particle, with a moment equal to that of the inertia forces of the particles, produces the same angular acceleration as for the separate particles. This radius is called the radius of gyration. In general,

The radius of gyration of a body rotating about an axis is the distance from the axis of rotation to a point at which the mass of the body may be considered to be concentrated; the location being such that the relation between the applied turning moment and the resultant angular acceleration is unchanged.

Let r_g = radius of gyration in feet of a rotating body,
M = mass of the body,
A = linear acceleration of a point located r_g feet from the center of rotation,
α = angular acceleration of the body, radians per second per second,
I_z = moment of inertia of the body about the axis of rotation.

If the mass of the body be considered to be concentrated at a distance r_g from the center, then the force F required to give it a linear acceleration A, is

$$F = MA = M \times r_g \times \alpha,$$

and the moment of this force is (multiplying both sides by r_g),

$$\overline{M} = F \times r_g = M \times r_g^2 \times \alpha.$$

By Art. 98, $\overline{M}$ also equals $I_z \times \alpha$. Hence

$$M \times r_g^2 \times \alpha = I_z \times \alpha.$$

Whence, $$I_z = Mr_g^2 \text{ and } r_g = \sqrt{I \div M}.$$

Certain problems involving the relation

$$\overline{M} = I_z \times \alpha,$$

are more readily solved by using the similar relation (from $I_z = Mr_g^2$)

$$\overline{M} = F \times L = M \times r_g^2 \times \alpha.$$

PROBLEM 248. A flywheel with a moment of inertia equal to 120 units of mass-feet squared is accelerated under a constant turning moment of 85 lb.-ft. What is is the angular acceleration? *Ans.* 0.708 radian per sec. per sec.

PROBLEM 249. Fig. 176 shows a rotor mounted on a shaft, weighing 1200 lbs., and having a radius of gyration = 2.15 ft. It is retarded by a constant friction force at the bearings, $\overline{F}$ = 88 lbs. acting at a distance from the axis of rotation of 1.5 ins. What is the retardation, neglecting the retardation due to windage? What time is required for the rotor to come to rest from a velocity of 1000 r.p.m.?

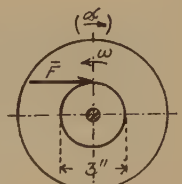


FIG. 176.

Ans. 1616 sec.

101. Problems of the type of 248 and 249 cannot be solved without a knowledge of the moment of inertia or of the radius of gyration and weight of the body under consideration. The former may be obtained experimentally or mathematically. Mathematical determinations depend on the application of calculus to the integral $\int r^2 dM$. Derivations of this quantity may be found in any standard work on calculus for all conventionally shaped solids, and therefore none will be included in this book.

Formulas, resulting from such derivations, give moments of inertia and radii of gyration referred to an axis of rotation *passing through the center of gravity of the rotating body* (*or shape*). Such an axis is called a centroidal axis. The position of the axis of rotation, or "reference axis," should be carefully noted when applying such formulas.

The procedure here adopted is to determine the radius of gyration of a mechanical part whose motion is to be studied, from which its moment of inertia may be found (from $I = M \times r_g^2$). It should be noted that the radius of gyration of a body is not dependent upon the density of the material of which the body is composed, as long as it is homogeneous, but only upon the distribution of its particles with reference to its axis of rotation, that is, upon the shape of the body and the location of the axis. (See Problems 246 and 247.)

102. Rectangular and polar moments of inertia. Consider first an area P, as shown by Fig. 177, and imagine it to be in reality a very thin plate *whose weight per unit of area* is W'. Then if dP represents a differential of area, the differential of mass of the plate is

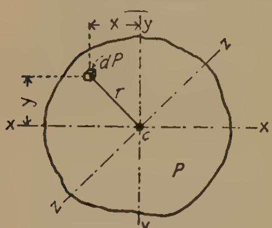


Fig. 177.

$$dM = \frac{W'dP}{32.2}.$$

Assume that this plate is revolving about an axis xx through its centroid, c. This motion causes every point of the surface to describe a circle, the plane of which is perpendicular to xx. The moment of inertia of the plate with reference to the axis xx is

$$\int \frac{W'dP}{32.2} \times y^2 = \frac{W'}{32.2}\int y^2 dP,$$

in which y is the distance from dP to the axis.

The expression included in the last integral is called the *rectangular moment of inertia of the area* with respect to the axis xx and will be denoted by I_x. Its units are inches or feet to the fourth power. A mathematical definition follows:

The rectangular moment of inertia of an area is the sum of the products of each differential area and the square of its distance from a straight line lying in its plane.

If the plate is rotated about the centroidal axis, yy, perpendicular to xx, its moment of inertia is the same as before, except that the dimension x is interchanged for y, and the moment of inertia of the area with respect to yy (called I_y) has the same form as when referred to the other axis.

Imagine now that the plate is revolved in the plane of xy, about a centroidal axis zz perpendicular to the plane xy, Fig. 177. To find the moment of inertia with respect to this axis it is necessary to find the sum of the products of the differential areas and the square of the distances r from the center of rotation. From the figure,

$$r^2 = x^2 + y^2$$

for every differential area. Hence the " polar " moment of inertia of the area is

$$\int r^2 dM = + \int (x^2 dM + y^2 dM) = I_z = I_x + I_y.$$

103. Radius of gyration of an area. Again considering Fig. 177 as a thin plate rotating about xx, its radius of gyration is

$$r_{g_x} = \sqrt{\frac{(W' \div 32.2) \times \int y^2 dP}{(W' \div 32.2) \times P}}$$

$$= \sqrt{I_x \div P}.$$

Similarly the radius of gyration referred to the yy axis

$$r_{g_y} = \sqrt{I_y \div P},$$

and to the zz axis is

$$r_{g_z} = \sqrt{I_z \div P} = \sqrt{(I_x + I_y) \div P} = \sqrt{r^2_{g_x} + r^2_{g_y}}.$$

Figure 178 represents a solid built up of thin plates, each of thickness dz, and of the same cross-section, perpendicular to zz, as the area of Fig. 177. Since the radius of gyration referred to zz as an axis of rotation, of any element, dz thick, is the same as that of any other, the radius of gyration of the solid is the same as that of any element. This leads to the following theorem:

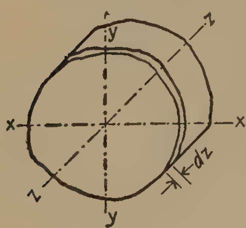


FIG. 178.

104. Theorem. *The radius of gyration of a solid generated by an elementary plane area passing in a direction perpendicular to the plane of the area, is the same as the radius of gyration of the generating area, provided that the axis to which the radius of gyration is referred is perpendicular to the plane of the generating area.*

It follows that the moment of inertia of a body having a form such as described in the preceding theorem equals its mass times the square of the radius of gyration of a cross-section of the body cut out by a plane perpendicular to the axis of rotation.

PROBLEM 250. In Fig. 179 two centroidal axes of a rectangle, xx and yy, are shown. $I_x = 9$ and $I_y = 16$ in.[4] What is the polar moment of inertia? What is the radius of gyration with respect to xx? With respect to yy?

Ans. 0.86, 1.153 ins., for radii of gyration.

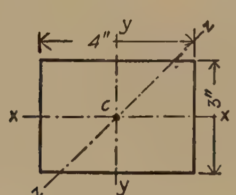


FIG. 179.

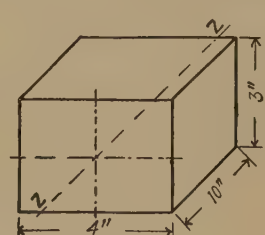


FIG. 180.

PROBLEM 251. What is the radius of gyration in the preceding problem with respect to zz?

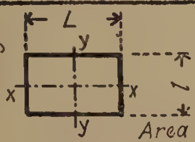
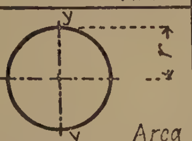
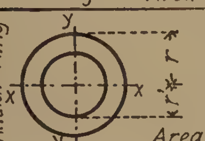
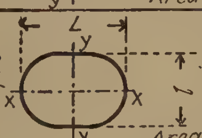
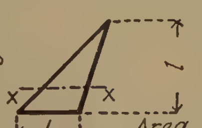
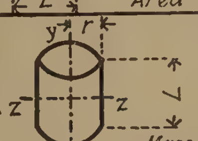
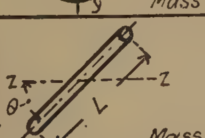
Ans. 1.45 ins.

PROBLEM 252. Fig. 180 represents a rectangular prism rotating about a centroidal axis zz. The material of the prism is cast iron weighing 0.26 lb. per cu. in. Using the data of the preceding problem, what is the mass moment of inertia of the prism?

Ans. 2.04 units of mass × in.[2]

105. Formulas for I and r_0 for conventional areas and for some solids are given on page 109. For each figure the reference axis is centroidal.

MOMENTS OF INERTIA AND RADII OF GYRATION
WITH RESPECT TO CENTROIDAL AXES

Figure	I_{xx}	I_{yy}	I_{zz}	$(r_g)_x$	$(r_g)_y$	$(r_g)_z$
Rectangle 	$\frac{Ll^3}{12}$	$\frac{lL^3}{12}$	$\frac{Ll(L^2+l^2)}{12}$	$\frac{l}{\sqrt{12}}$	$\frac{L}{\sqrt{12}}$	$\frac{\sqrt{L^2+l^2}}{\sqrt{12}}$
Circle 	$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{2}$	$\frac{r}{2}$	$\frac{r}{2}$	$\frac{r}{\sqrt{2}}$
Anular Ring 	$\frac{\pi(r^4-r'^4)}{4}$			$\frac{\sqrt{r^2+r'^2}}{2}$		
Elipse 	$\frac{\pi L l^3}{4}$	$\frac{\pi l L^3}{4}$	$\frac{\pi L l(L^2+l^2)}{4}$	$\frac{l}{2}$	$\frac{l}{2}$	$\frac{\sqrt{L^2+l^2}}{2}$
Triangle 	$\frac{Ll^3}{36}$			$\frac{l}{3\sqrt{2}}$		
Cylinder 		$\frac{1}{2}Mr^2$	$M\left(\frac{r^2}{4}+\frac{L^2}{12}\right)$		$\frac{r}{\sqrt{2}}$	$\frac{\sqrt{r^2+\frac{L^2}{3}}}{2}$
Slender Rod 			$\frac{1}{12}ML^2\sin^2\theta$. When $\theta = 90°$ $\frac{1}{12}ML^2$			$\frac{L}{\sqrt{12}}\sin\theta$ When $\theta = 90°$ $\frac{L}{\sqrt{12}}$

The formula of one value of I for each figure is obtained by integral calculus, from which the remaining values are deduced by algebra.

It is worth noting that, although the moment of inertia of an area appears to be a purely mathematical conception, it has an important physical application in the "mechanics of materials." The moment of inertia of a cross-sectional area of a beam is a measure of the resistance of the beam against bending. It must be known in order to design a beam to carry a stated load.

For the following problems start with the value for I_x in the table on page 109 for the stated figure and deduce the values for I_y, I_z and the radii of gyration.

PROBLEM 253. For the rectangle.
PROBLEM 254. For a square of side L.
PROBLEM 255. For the circle.

106. Transfer of I and r_g to a non-centroidal axis. Let the irregular outline of Fig. 181 represent any area of P units. Given that its moment of inertia about the centroidal axis xx is I_x, to find the moment, I_x', about a parallel axis $x'x'$ in the plane of the figure and distant from xx by the distance L.

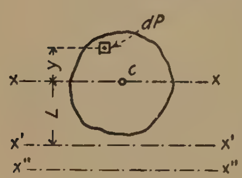


FIG. 181.

The square of the distance of any differential area dP from $x'x'$ is now $(y + L)^2$ or $(L - y)^2$. Hence the required value

$$I_x' = \int (y^2 \pm 2Ly + L^2)dP$$

$$= \int (y^2dP + L^2dP).$$

Since $\int 2LydP = 0,$

$$I_x' = I_x + PL^2.$$

The term $\int 2LydP$ equals zero because the plus and minus values of y exactly equal each other and go with balancing differential areas as they are referred to a centroidal axis, that is, an axis of symmetry.

The radius of gyration referred to x' is found by dividing the preceding expression for I'_x by the area P and extracting the square root:

$$r_{gx}' = \sqrt{r_{gx}^2 + L^2}.$$

PROBLEM 256. Find the value of I_x' and r_g' with reference to an axis 2 ins. below xx and parallel to it, of the rectangle of Fig. 179.

Solution.

$$I_x = 4 \times 27 \div 12 = 9 \text{ ins.}^4,$$

$$I_x' = 9 + 12 \times 2^2 = 57 \text{ ins.}^4,$$

$$r_{gx}' = \sqrt{57 \div 12} = 2.18 \text{ ins.}$$

A similar relation may be proved for solids. If Fig. 181 represents the outline of a body of mass M (instead of an area of P units) revolving about xx as an axis, its moment of inertia with respect to $x'x'$ is

$$I_x' = I_x + ML^2,$$

and the radii of gyration with respect to these axes bear the same relation to each other as those for areas, namely.

$$r_{g_x}' = \sqrt{\frac{I_x'}{M}} = \sqrt{r^2_{g_x} + L^2},$$

and similarly for r_g referred to the yy or zz axes.

PROBLEM 257. Find I_z' for the area of Fig. 179 for an axis located 2 ins. below zz and parallel to it. Find r_{g_z}'. *Ans.* 73 ins.^{4}, 2.5 ins.

PROBLEM 258. If the solid shown in Fig. 180 is of cast iron weighing 0.26 lb. per cu. in., and it rotates about an axis $z'z'$, 2 ins. below zz and parallel to it, what is r_{g_z}'? Find the moment of inertia of the solid about $z'z'$ in units of mass $\times$ inches^2. *Ans.* 2.5 ins., 5.9 units of mass $\times$ ins.^{2}.

PROBLEM 259. Find the moment of inertia of a circular plate of cast iron 26 ins. in diameter, weighing 0.1 lb. per sq. in. of area if it rotates about a centroidal axis perpendicular to its face. (Suggestion: Find r_{g_z} first, then use $I_z = M \times r_{g_z}^2$.) *Ans.* 139 units of mass $\times$ ins.^{2}.

PROBLEM 260. Find the moment of inertia of the circular plate of Problem 259, if the axis of rotation is perpendicular to the face and passes through a point on its rim. *Ans.* 417 units of mass $\times$ ins.^{2}.

The student is especially cautioned that the transfer formulas apply only from a *centroidal axis* to a parallel axis. Thus, if in Fig. 181, I_x' is known and it is required to find I_x'', it is first necessary to find I_x and then use this value of I_x in the transfer formula to determine I_x''.

FIG. 182.

107. I and r_g of Composite Areas and Solids. Figure 182 represents an area composed of two areas, P and Q, whose moments of inertia, I_x and I_x', respectively,

referred to the axis xx, are known. Then the moment of inertia of the whole area with respect to xx is

$$I_x + I_x',$$

and the radius of gyration is

$$\sqrt{(I_x + I_x') \div (P + Q)}.$$

It should be noted that before moments of inertia may be added as above, they must be *referred to the same axis.*

If Fig. 182 represents the outline of two solids instead of two areas, similar relations are true.

The principles outlined above may now be applied to practical problems.

PROBLEM 261. Find the radius of gyration of the shaded area of Fig. 183, with respect to the centroidal axis zz, perpendicular to the plane of the area.

Solution. I_z of the circle $= 3.14 \times 5^4 \div 2 = 984$ ins.^{4}

I_z' of one rectangle (centroidal) $= 4 \times 3(4^2 + 3^2) \div 12 = 25$ ins.^{4}.

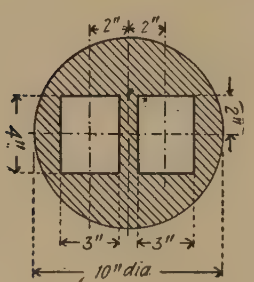


FIG. 183.

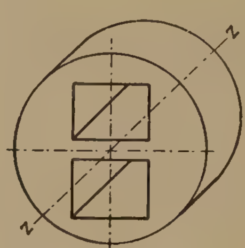


FIG. 184.

Transfer I_z' from axis $z'z'$ to axis zz.

I_z for one rectangle $= 25 + 2^2 \times 12 = 73$ ins.^{4}.

Combined moment with respect to zz of the circle minus two rectangles

$$= 984 - 73 - 73 = 838 \text{ ins.}^4.$$

Area of composite figure $= 78.54 - 12 - 12 = 54.5$ ins.^{2}.

r_{gz} of combined figure $= \sqrt{838 \div 54.5} = 3.91$ ins.

PROBLEM 262. Fig. 184 represents a cylindrical solid with two rectangluar holes passing through it. Any cross-section of Fig. 184 has the same dimensions

as Fig. 183. What is the moment of inertia of the solid with respect to zz, if it weighs 100 lbs.?

Solution. The radius of gyration of this solid is the same as that of the area of Fig. 183, which was found in the preceding problem to be 3.91 ins. Hence, for the solid

$$I_z = M \times r_{g_z}^2 = (100 \div 32.2) \times 3.91^2 = 47.5 \text{ units of mass} \times \text{ins.}^2,$$

and

$$I_z = 47.5 \div 144 = 0.33 \text{ units of mass} \times \text{ft.}^2$$

PROBLEM 263. Deduce the formulas for the annulus from the value of I_z given in the formulas on page 109.

Figure 185 is to be referred to in the following problems. This represents a flywheel, cast solid, and composed of a rim and hub, each rectangular in section, and six arms, each of which may be considered a slender rod. The rim weighs 2200 lbs., the hub 300 lbs., and the arms 250 lbs. each.

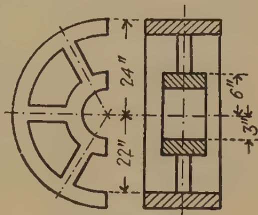


FIG. 185.

PROBLEM 264. What is the moment of inertia in units of mass $\times$ ft.^{2} with respect to the axis zz, of the flywheel rim? *Ans.* 252.

PROBLEM 265. Of the hub? *Ans.* 1.46.

PROBLEM 266. Of one arm, referred to the centroidal axis of the arm parallel to the axis zz? (See p. 109, $\theta = 90°$.) *Ans.* 1.15.

PROBLEM 267. Of the whole flywheel, referred to zz? *Ans.* 323.9.

PROBLEM 268. How many radians per second per second acceleration will be caused if a turning moment of 100 lb.-ft. be applied to the rim? (See Art. 98.) *Ans.* 0.31.

PROBLEM 269. What is the radius of gyration of the flywheel in feet? *Ans.* 1.61.

PROBLEM 270. Assume the mass of the flywheel to be concentrated at a distance from its center equal to the answer of Problem 269, and to revolve in a circle of this radius; and that the tangential force F applied to the concentrated mass produces a moment of 100 lb.-ft. (as in Problem 268). F equals 100 lb.-ft. divided by the radius of gyration. Using $F = MA$, find A and show that this is the same as produced by the angular acceleration found in Problem 268.

PROBLEM 271. Suppose that Fig. 185 may have any proportions, in regard to the rim, arm and hub sizes. Is it possible that the radius of gyration of the flywheel as a whole be

(*a*) Greater than the outside radius of the rim,
(*b*) Greater than the inside radius of the rim,
(*c*) Less than the inside radius of the rim,
(*d*) Less than the outside radius of the hub?

108. Curvilinear Motion. If a body is accelerated through a curved path, not the arc of a circle, the radius of curvature varies along the path.

If the instantaneous values of this radius, and of the linear velocity and acceleration at any position be used in the equations for radial and tangential velocities and acceleration and corresponding forces, instead of the constant radius r, and values of V and A, all equations in this section become true for curvilinear motion in general as well as for rotation; the values obtained changing from instant to instant.

Since this sort of motion is unusual in engineering problems, it will not be dealt with in this book.

PART XIII

WORK, ENERGY AND POWER

109. Work, in a mechanical sense, is the effect produced when a body is moved from one position to another by an applied force, or against a resisting force. Numerically it is equal to the product of the force by the distance through which the body is moved in the direction of the force. If the force varies in magnitude during the motion, its average value may be used.

The units of work, in this book, are feet $\times$ pounds, called foot-pounds, and will be designated by $\overline{W} = LF$. This should be distinguished from $\overline{M} = FL$, denoting moment in pound-feet.

The applied force may be a single one or the resultant of a number of forces. The product of the applied force and the distance through which it passes in its own direction will be called the " input," or " work supplied " to the machine.

The resisting forces fall into two classes: first, those such as friction, which must be overcome to accomplish the desired effect, and, second, those forces whose displacements constitute the desired effect. The work necessary to overcome friction, etc., will be called " lost work," and that expended in accomplishing the desired effect will be called " useful work."

For example, in the cylinder of a steam hoisting engine, the expansive force of the steam pushes the piston through a rectilinear distance. This is the input work. Geared to the engine shaft, there is a cable drum, the whole being arranged to lift material. The force of gravity on the material times the vertical distance lifted is the useful work. This is considerably less than the supplied work, owing to friction and other resistances. The difference is the lost work.

Care should be taken in the definition of " useful work " since all mechanical work is ultimately dissipated by friction to reappear as heat. For example, in the mechanism of an automobile, the work delivered by the

engine shaft is the useful work of the engine, but not of the car. Much of this is lost in transmission, and the tractive effort of the wheels may be said to be the useful work of the engine. This, however, is dissipated in overcoming friction at the front wheel bearings, windage against the car, flexing of tires, and so on, all of which reappear as heat.

Problem 272. Fig. 186 represents a block weighing 100 lbs. which is caused to slide uniformly up a plane as shown, from m to n, without frictional resistance. What is the useful work? Calculate the input work of the applied force (a) if it acts horizontally and (b) if it acts parallel to the plane.

Solution. The useful effect is the raising of the body against gravity. The displacement of the body in the direction of this force is 30 ft. The useful work is therefore 30 × 100 = 3000 ft.-lbs.

Figs. 186 and 187 show the space and force diagrams if the applied force F is horizontal. By similar triangles $F = 100 \times 30 \div 40 = 75$ lbs. The distance

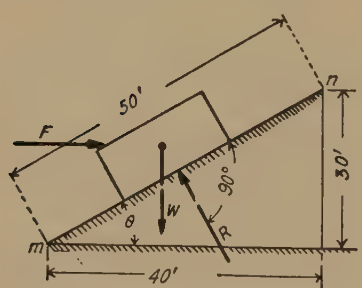


Fig. 186.

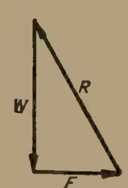


Fig. 187.

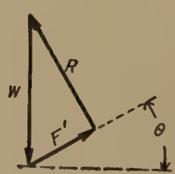


Fig. 188.

through which the body is displaced in the direction of F is 40 ft. Hence, the input is 40 × 75 = 3000 ft.-lbs. If the applied force, F', is parallel to the plane, the force diagram is as in Fig. 188. By similar triangles, $F' = 100 \times 30 \div 50 = 60$ lbs. The distance in the direction of F' is 50 ft. Hence the input work equals 50 × 60 = 3000 ft.-lbs.

In this problem, since it is assumed that there is no friction, the input work equals the useful work and the operation is 100 per cent efficient, a result that could not be obtained in any real mechanical operation.

Problem 273. A rectangular homogeneous block weighing 250 lbs. is turned around its right-hand lower corner as shown in Fig. 189 to the position shown in Fig. 190. How much useful work is done? *Ans.* 250 ft.-lbs.

PROBLEM 274. Same as Problem 272, except that friction is to be considered, with $f' = 0.1$. Find also the lost work when the applied force is horizontal and when it is parallel to the inclined plane. Check by applying the equation

$$\text{Input Work} = \text{Lost Work} + \text{Useful Work}.$$

Ans. 3680, 680, 3000 ft.-lbs.; 3400, 400, 3000 ft.-lbs.

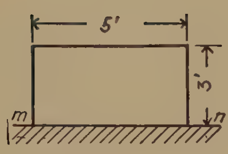


FIG. 189.

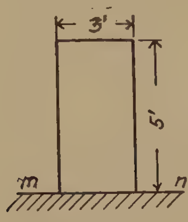


FIG. 190.

PROBLEM 275. The draw-bar pull from a locomotive pulling a 125-ton train up a 1 per cent grade is 3750 lbs. How much work is expended by the locomotive in a distance of 1 mile? How much work is expended against gravity?

Ans. 19,850,000 ft.-lbs.; 13,200,000 ft.-lbs.

PROBLEM 276. An aeroplane, travelling at 120 miles per hour at a constant level, has a horizontal pull at the propeller shaft of 500 lbs. How much work is done by the propeller shaft in one minute? *Ans.* 5,280,000 ft.-lbs.

110. Efficiency. The ratio of the useful work to the input work is called the mechanical efficiency. This may be put in the following form:

$$\begin{aligned}\text{Mech. Eff.} &= \text{useful work} \div \text{input work}\\ &= (\text{input work} - \text{lost work}) \div \text{input work}\\ &= 1 - \text{lost work} \div \text{input work}.\end{aligned}$$

PROBLEM 277. Using the answers to Problem 274, what is the efficiency when the force is horizontal? When it is parallel to the inclined plane?

Ans. 81.5 per cent, 88.3 per cent.

111. Work delivered by rotating shafts. Torque. Figure 191 represents a shaft supported by bearings. At one end of the shaft there is a crank r feet in radius. A force, F, is applied to the center of the crank pin and in a direction tangential to the circular path of the crank-pin center. If there is some resisting effort at the other end of the shaft, such as would result from a pulley and belt load, there may be uniform motion. Considering that F acts tangentially at all positions of the crank, then in one rev-

olution this force displaces the crank center through a path equal to $2 \times 3.14 \times r = L$. The work done in one revolution is

$$\overline{W} = LF = 2 \times 3.14 \times rF.$$

In this expression rF is called the "*torque*." Its units are the same as those of the moment of a force, or "pound-feet," "pound-inch," etc. In this book it will be symbolized in the same way as "moment."

$$\overline{M} = Fr.$$

Work supplied when a body is displaced in a circular path may be utilized in the displacing of another body in a rectilinear path or vice versa. In either case the work input equals the useful work plus the lost work.

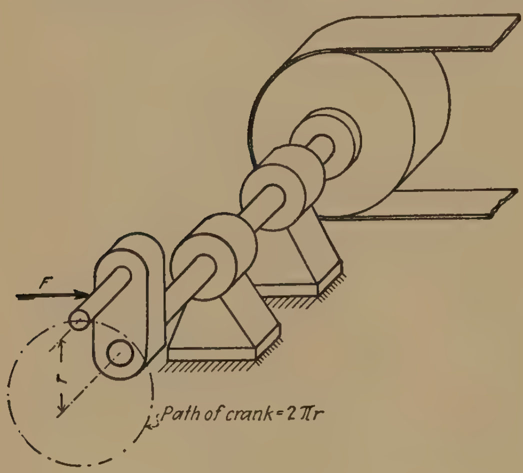


Fig. 191.

Problem 278. Suppose that the right-hand end of the shaft shown in Fig. 191 is supplied with a cable drum as in Fig. 192, the end of the cable supporting a body weighing 500 lbs., the radius of the crank being 0.5 ft., and of the drum 2 ft. How much work is supplied at the crank when the drum is turned at a uniform rate through two revolutions, if the friction at the bearings is negligible? What is the torque and the magnitude of the force, F, at the crank?

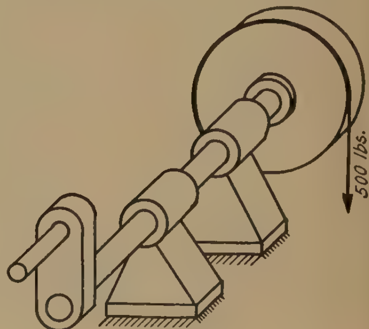


Fig. 192.

Solution. Since the friction at the bearings is inconsiderable, the torque produced by W = the torque produced by F, and

$$\overline{M} = F \times 0.5 = 500 \times 2 = 1000 \text{ lb.-ft.}$$
$$F = 2000 \text{ lbs.}$$

$$\text{Work input in two revolutions} = 2(2 \times 3.14 \times \overline{M})$$
$$= 2 \times 2 \times 3.14 \times 1000$$
$$= 12{,}560 \text{ ft.-lbs.}$$

The last answer can be obtained by another method. Thus: In two revolutions the body will be raised a height equal to two circumferences of the drum or $2 \times 2 \times 3.14 \times 2 = 25.12$ ft. Since there is no lost work,

$$\text{Work input} = \text{useful work} = 25.12 \times 500$$
$$= 12{,}560 \text{ ft.-lbs.}$$

PROBLEM 279. Using the data of the preceding problem, except that friction at the bearings causes 1000 ft.-lbs. of lost work per revolution, how much work is done at the crank in one revolution? What force is necessary at the crank, and what torque is there at the crank and at the drum?

Ans. 7280 ft.-lbs., 2321 lbs., 1160 lb.-ft. 1000 lb.-ft.

When rotative effort is transmitted by a shaft, it is not always specified at what radius the driving force is applied or at what radius the delivered force is transmitted. This is confusing to the student, until he gets a clear understanding that the work transmitted does not depend upon the magnitude of the force alone, or on the radius alone, but on the product of the two, namely, the torque. Thus if the torque, $\overline{M} = Fr$, is known in pound-feet, the work, $2 \times 3.14 \times \overline{M}$ may be delivered at a small value of r (and a correspondingly small value of circumferential path, $2 \times 3.14 \times r$), in which case F is large, or at a large value of r, in which case F is small. In each case the work delivered is the same, provided the product of the force and the radius is the same.

The following definition of torque helps to a more concrete understanding:

Torque is, in magnitude, that force which, if applied at unit distance from the axis of a body under rotation, or tending to rotate, would accomplish, or tend to accomplish, the same rotative effects as the forces actually applied.

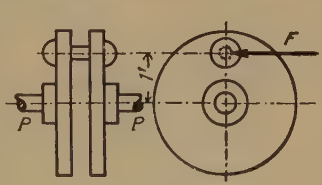


FIG. 193.

Figure 193 represents a continuous shaft, P, transmitting work under a torque of $\overline{M}$ pound-feet. Imagine the shaft to be cut and a coupling inserted, as shown. The turning effort is now transmitted by a force, F, acting through a single bolt located 1 foot from the shaft axis. The torque is $F \times 1 \text{ ft.} = F$, numerically, and the circumferential path of F is $2 \times 3.14 \times 1$ ft. If the

student has difficulty in understanding rotative work, he may use the above definition and represent the existing torque by a tangential force equal to the torque in magnitude, and acting in a circular path of 1 foot radius, or 6.28 feet in length, per revolution.

PROBLEM 280. A shaft revolving at 80 r.p.m.is under a torque of 1250 lb.-ft. How much work will it transmit in 90 seconds? *Ans.* 940,000 ft.-lbs.

PROBLEM 281. If all the work transmitted by the shaft of Problem 280 is utilized in raising a body weighing 1000 lbs., how many feet will the body be raised in 1 second? *Ans.* 10.44 ft.

PROBLEM 282. An automobile engine of popular make has four cylinders as shown in Fig. 157, each containing a piston with a 4-in. stroke. Each piston makes one downward and one upward stroke per revolution. Work is done by the expansive effort of the gas on alternate downward strokes of each piston, so that for the four cylinders there are two working strokes per revolution of the main shaft. If the average effective force on each piston during the working stroke is 900 lbs., and if 25 per cent of the work done in the cylinder is lost, what is the torque at the shaft? *Ans.* 71.8 lb.-ft.

112. Relation between torque and rotative speed. Figure 194 represents a driving shaft, P, and a follower shaft, Q, so connected by gears or other mechanism that P revolves at a speed different from that of Q.

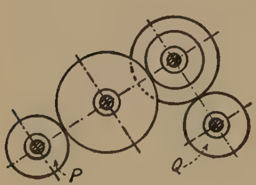


FIG. 194.

Let $\overline{M}_p$ and $\overline{M}_q$ represent the torque of P and Q respectively.

N_p and N_q represent the revolutions per minute of P and Q respectively.

If the lost work is neglected, the work delivered by the driver, P, in one minute equals the work received by the follower, Q, in the same time, and

$$2 \times 3.14 \times \overline{M}_p \times N_p = 2 \times 3.14 \times \overline{M}_q \times N_q,$$

$$\overline{M}_p \div \overline{M}_q = N_q \div N_p$$

that is, the torques are inversely proportional to the rotative speeds.

If friction losses are taken into account, the follower torque is lessened by a corresponding amount. If k stands for the appropriate ratio of rotative speeds, and E for the efficiency of the operation, expressed as a fraction of the input work, then,

$$\text{Torque of follower} = E \times k \times \text{Torque of driver.}$$

PROBLEM 283. A shaft P drives another shaft Q by means of a belt. The belt pulley on P is 9 ins. in diameter and on Q is 36 ins. If the torque of P is 300 lb.-ft., what is the torque of Q if there are no losses? *Ans.* 1200 lb.-ft.

PROBLEM 284. What is the value of k in Problem 283?

PROBLEM 285. Same as Problem 283 if belt losses are 5 per cent of the input work. *Ans.* 1140 lb.-ft.

PROBLEM 286. Using the data of Problem 283, if the part of the belt leading off the drive pulley is under a tension of 150 lbs., what is the tension in the part of the belt leading onto the pulley? (NOTE: The torque of P = 300 lb.-ft. = the difference between the two torques effected by the two belt pulls.) *Ans.* 950 lbs.

PROBLEM 287. The automobile of Problem 282 has a gear ratio of $3\frac{7}{11}$ to one. That is, the engine makes $3\frac{7}{11}$ revolutions for each revolution of the rear axles. If the efficiency of transmission is 80 per cent, what is the torque at the rear axles? *Ans.* 209 lb.-ft.

PROBLEM 288. If in Problem 287 the diameter of the rear wheels is 30 ins., what is the tractive force? *Ans.* 167 lbs. for both wheels.

113. Work done by a variable force. This can be shown graphically as in Fig. 195, in which the variable magnitude is shown by the ordinates of the curve and the rectilinear displacement by its abscissæ. The work done for a differential of displacement is

$$FdL,$$

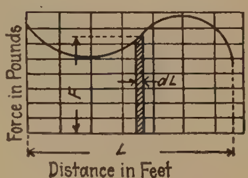


FIG. 195.

which may be integrated when the law of variation between F and L is known. For example, if the curve is that of a rectangular hyperbola, $F \times L$ = constant, and

$$FdL = \text{a constant} \times \int_{L}^{L'} dL \div L$$

$$= FL \log_e(L' \div L).$$

NOTE. (Log to the base $e = 2.3 \times$ log to the base 10, approximately.)

The area under the curve, in scale units, equals the work done in foot-pounds. Hence, in Fig. 195, if the curve cannot be expressed by a mathematical relation, the work may be calculated from the area, in scale units, under the curve.

Mean height of area, lbs., $\times$ length of area, L, in feet, that is

Work = average force times total displacement.

The mean height in this case is first obtained by the arithmetical averaging of a number of equidistant coordinates or by the use of the instrument known as the "planimeter." The resulting value in inches is multiplied by the vertical scale.

PROBLEM 289. A spring is 8 ins. long when free from external force. A force of 60 lbs. is required to compress it 1 in., the force required being directly propor-

tional to the amount of linear compression. Make a graph similar to Fig. 195 to show the variation of the force required when the spring is changed in length from 8 to 6 ins. How much work is done? How much work is done when the spring is compressed from 6 to 5 ins.? *Ans.* 120, 150 in.-lbs.

PROBLEM 290. It is assumed that the action of the working medium in an engine cylinder is such that $F \times L$ is a constant, F being the total force behind the piston and L the displacement of the piston from dead center. How much work is done when the piston passes from a position of $L = 1$ ft. to $L' = 2$ ft., the value of F at the first position being 7800 lbs.? Make a graph of this and shade the area representing work. (Use formula $W = FL \log_e (L' \div L)$, in which FL, being constant, may be taken at any value of L.)

114. Potential and kinetic energy. Mechanical energy is the capacity of a body to do work by virtue, first, of its position, or, second, of its motion. The first is called " potential " and the second " kinetic " energy.

If a body weighing W pounds is elevated to a height of L feet above the level of the earth, and then allowed to descend at a uniform velocity, V, against a uniform resistance, the work done (barring losses) in overcoming the resistance is LW foot-pounds. At the upper level the body is said to have a potential energy of LW foot-pounds.

If the body in the preceding paragraph be allowed to fall freely from rest, air resistances and other disturbances being removed, after descending L feet its velocity will have been accelerated to a certain value depending upon the time during which the acceleration is in effect, or upon L. The potential energy is then converted into the kinetic form. Mechanical work may be obtained from kinetic energy by the interposition of some properly designed machine and, were it not for mechanical imperfections resulting in lost work, the kinetic energy could be completely converted into useful work.

The units of kinetic or of potential energy are the same as of work, since the one equals the other. Kinetic energy, in this book, is designated by the letter K, for rectilinear motion, and by K', for rotary motion.

115. Kinetic energy of translation. A notion of the physical aspect of kinetic energy may be had from the following. If a body is travelling in a rectilinear path at a certain velocity, then brought to rest uniformly, the inertia force, which is in the direction of the motion, may do work upon an interposed body, moving in the same direction, until the velocity of the first body is zero. The product of the displacement, L, and F_i, the inertia force ($= MA$), during its operation, until $V = 0$, is the kinetic energy. Conversely, if a body is accelerated from zero to a given velocity, its kinetic energy at the given velocity equals the work done by the force required for acceleration.

Following is the derivation of the mathematical expression for kinetic energy in terms of the velocity and mass.

Let F_i = inertia force, in pounds, under a negative acceleration of a body of mass M,
A = retardation in feet per second per second,
V = initial velocity of the body,
T = time in seconds required to reduce velocity from V to zero,
L = distance traversed during T seconds.

Then
$$F_i = MA.$$

Multiply both sides by L. The work done then is

$$LF_i = LMA.$$

Substituting for A its value, $V \div T$,

$$LF_i = LMV \div T.$$

Now $L \div T$ = the average velocity, or $V \div 2$, and, by definition, $LF_i = K$.

Therefore,
$$K = MV^2 \div 2 = WV^2 \div 64.4.$$

116. Problems involving work and kinetic energy. Let V_1 be the velocity of a body at the beginning of a mechanical process, and V_2 that at the end, in feet per second. Then, if $\overline{W}$ is the input work, $\overline{W}'$ the lost work, and $\overline{W}''$ the useful work,

$$\overline{W} + WV_1^2 \div 64.4 = \overline{W}' + \overline{W}'' + WV_2^2 \div 64.4.$$

This is an energy equation stating that *the input work plus the kinetic energy entering into such a transaction equals the useful work plus losses plus kinetic energy at the end of the process.* If there is a difference in potential energy involved, this could be taken into account by adding to the left and right-hand side of the equation the item, weight times distance above some datum plane, at the beginning and at the end of the process, respectively.

In the energy equation, if the lost work equals zero, and if there is no useful work,

$$\overline{W} = W(V_2^2 - V_1^2) \div 64.4,$$

or, the input work is expended to increase the kinetic energy from an

amount corresponding to the initial velocity to that corresponding to a larger final velocity.

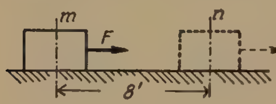


Fig. 196.

Problem 291. It is required to increase the velocity of a body weighing 100 lbs. as shown in Fig. 196, from 10 ft. per sec. in position m, to 20 ft. per sec. in position n. Assuming that there is no frictional resistance, what is the magnitude of the force F?

Solution. Kinetic energy at $m = 100 \times 10^2 \div 64.4 = 155$ ft.-lbs.

Kinetic energy at $n = 100 \times 20^2 \div 64.4 = 620$ ft.-lbs.

Input work $= \overline{W} = 620 - 155 = 465$ ft.-lbs.

Since $L = 8$ ft., $F = 465 \div 8 = 58.1$ lbs.

Problem 292. If the plane in Fig. 196 is slanted upward 30°, F remaining parallel to the plane, and if the coefficient of kinetic friction is 0.09, how much work is required to move the body along the plane a distance of 8 ft.? $V_1 = 10$ ft. per sec. and $V_2 = 20$ ft. per sec.

Solution. Friction $= 0.09 \times (100 \cos 30°) = 7.8$ lbs.
Lost work $= 8$ ft. $\times$ 7.8 lbs. $= 62.4$ ft.-lbs.
Work expended in raising body vertically $= (8 \sin 30°) \times 100 = 400$ ft.-lbs.
Increase in kinetic energy (from Prob. 291) $= 465$ ft.-lbs.
Input work $= 62.4 + 400 + 465 = 927.4$ ft.-lbs.

Problem 293. A body falls freely (without wind resistance, etc.) under the action of gravity a distance of 100 ft. Using potential and kinetic energy relations, calculate the velocity of the body at the end of the fall. *Ans.* 80.3 ft. per sec.

Problem 294. An 1800-lb. automobile travelling at 20 miles per hour is retarded by a constant force of 267 lbs. acting parallel to the motion. In what distance can the car be brought to rest? *Ans.* 89.9 ft.

Problem 295. A four-wheel brake car weighing W lbs. has a velocity of 20 miles per hour. In what distance can it be brought to rest if the coefficient of friction between wheels and roadway is 0.667? *Ans.* 20 ft.

Problem 296. An 1800-lb. automobile is travelling on a level road at 40 miles per hour, under a constant tractive force at the rear wheels of 167 lbs. It strikes a 12 per cent grade. Assuming the tractive force to remain constant, and that the various resistances are constant and equal to 25 lbs., parallel to the motion, what will be the velocity of the car at a distance of 1000 ft. up the grade? (Suggestion: write energy equation.) *Ans.* 19 miles per hour.

117. Kinetic energy of rotating bodies. The mathematical expression is parallel to that for translation, the moment of inertia, I_z, appearing instead of mass, M, and angular velocity ω, in radians per second, instead of linear velocity, V. Thus, if K' is the kinetic energy of a rotating mass, at velocity, ω

$$K' = I_z \times \omega^2 \div 2.$$

To prove this, consider a differential mass, dM, of a rotating body, the distance of dM from the axis of rotation being r feet. Then the linear velocity of dM is $V = \omega r$. Substituting this in the equation for the kinetic energy of translation, $K = MV^2 \div 2$, there is obtained the integral form for kinetic energy of rotation,

$$K' = \int (\omega^2 r^2 \div 2) dM = (\omega^2 \div 2) \int r^2 dM.$$

Since $\int r^2 dM = I_z$,

$$K' = I_z \omega^2 \div 2.$$

Since $$I_z = M \times r_{g_z}^2,$$

$$K' = M \times r_{g_z}^2 \times \omega^2 \div 2$$

$$= M \times V_g^2 \div 2,$$

in which $V_g = r_{g_z}\omega$ and is, therefore, the velocity of a point on the body located at a distance from the axis of rotation equal to the radius of gyration. That is, the kinetic energy of a rotating body is the same as that of translation of a concentrated mass, equal to the mass of the body, along a circular path whose radius is equal to the radius of gyration of the body.

PROBLEM 297. A flywheel weighing 2000 lbs. has a radius of gyration of 2 ft. If it is turning with a velocity of 60 radians per second, what is its kinetic energy?

Solution. The moment of inertia of the flywheel with respect to the axis of rotation $= I_z = (2000 \div 32.2) \times 2^2 = 248$ units of mass $\times$ ft.^{2},

$$K' = 248 \times 60^2 \div 2 = 447{,}000 \text{ ft.-lbs.}$$

PROBLEM 298. Check the above by assuming the mass to be concentrated at a point distant from the axis of rotation by an amount equal to the radius of gyration, and that its linear velocity is that produced by the angular velocity of Problem 297.

Solution. Velocity = 60 radians per second $\times$ 2 ft. = 120 ft. per sec.
Mass = $2000 \div 32.2 = 62.1$
$K' = 62.1 \times 120^2 \div 2 = 447{,}000$ ft.-lbs.

PROBLEM 299. Using the data upon the flywheel of Problems 264–269, Art. 107, find its kinetic energy at 250 r.p.m. *Ans.* 111,000 ft.-lbs.

PROBLEM 300. What is the change in kinetic energy of the flywheel of the preceding problem, if the revolutions per minute is reduced from 260 to 240?
Ans. 17,800 ft.-lbs.

PROBLEM 301. A flywheel having a weight of 185 lbs. has a radius of gyration of 7.5 ins. If it is turning at 1000 r.p.m., what is the linear velocity of a point on it 7.5 ins. from the center? What is the kinetic energy of the flywheel at this speed?
Ans. 65.4 ft. per sec., 12,330 ft.-lbs.

118. Problems involving work, torque and kinetic energy of rotating Bodies. When a body increases in rotative speed, the increase in kinetic energy equals the work done by the torque causing the increase. If the body decreases in rotative speed, the decrease in kinetic energy equals the useful work done plus the lost work. An energy equation may be written parallel to that of Art. 116.

$$2 \times 3.14\overline{M}N' + \frac{W\omega_1^2}{64.4} \times r_{g_2}^2 = 2 \times 3.14\overline{M}'N' + 2 \times 3.14\overline{M}''N' + \frac{W\omega_2^2}{64.4} \times r_{g_2}^2,$$

in which N' is the number of turns during which the change in velocity occurs and the expressions containing $\overline{M}$, $\overline{M}'$ and $\overline{M}''$ signify the input work, work utilized and lost, respectively; ω_1 and ω_2 denote the angular velocities at the beginning and end of the process, respectively. This equation may be arranged thus:

$$2 \times 3.14 \times N'(\overline{M} - \overline{M}' - \overline{M}'') = \frac{W}{64.4} \times (\omega_1^2 - \omega_2^2)r_{g_2}^2,$$

That is, the difference between the input work and the lost plus useful work, during N' turns, equals the change of kinetic energy of a rotating body during N' turns.

FIG. 197.

PROBLEM 302. A flywheel weighing 2000 lbs. has a radius of gyration of 2 ft. It is increased in velocity from 60 radians per second to 120 radians per second by an active force, F, applied as shown in Fig. 197. What input torque is necessary to make this change in velocity in 100 turns, if windage and frictional resistances cause a constant torque, $\overline{M}''$, of 159 lb.-ft.?

Solution. K' at 60 radians per sec.	= 447,000 ft.-lbs.
K' at 120 radians per sec.	= 1,788,000 ft.-lbs.
Increase in K'	= 1,341,000 ft.-lbs.
Work required to overcome resistances	= 2 × 3.14 × 159 × 100
	= 100,000 ft.-lbs.
Therefore, total work required	= 1,441,000 ft.-lbs.,

whence $\overline{M} = 1{,}441{,}000 \div (2 \times 3.14 \times 100) = 2300$ lb.-ft.

PROBLEM 303. A flywheel with a radius of gyration of 1.6 ft., weighing 4000 lbs., changes in speed from 260 to 240 r.p.m. in $\frac{1}{4}$ turn. How many foot-pounds of work will be delivered by the shaft, not allowing for losses? *Ans.* 17,800 ft.-lbs.

PROBLEM 304. Allowing in the preceding, losses equal to 10 per cent of the kinetic energy change when the speed changes from 260 to 240 r.p.m., what torque is delivered due to this change in velocity? How much useful work is done during the $\frac{1}{4}$ turn required for the change in velocity? *Ans.* 10,200 lb.-ft.,16,020 ft.-lbs.

PROBLEM 305. A flywheel with radius of gyration of 1.5 ft. is required to deliver 10,000 ft.-lbs. of work when its speed varies no more than 5 per cent above and 5 per cent below an average of 240 r.p.m. What must be the weight of the flywheel? *Ans.* 2260 lbs.

***119.* Weight of flywheel required for speed regulation.** In the operation of a reciprocating steam or gas engine, the force delivered by the working fluid in the cylinder varies throughout the stroke, as also does the inertia force of the reciprocating parts. The algebraic sum between these two forces is transmitted through the piston and connecting rod to the crank pin. The thrust of the connecting rod on the crank pin is tangential to the crank circle only in two positions of the crank; at other positions the tangential force at the crank is a component of the connecting rod thrust

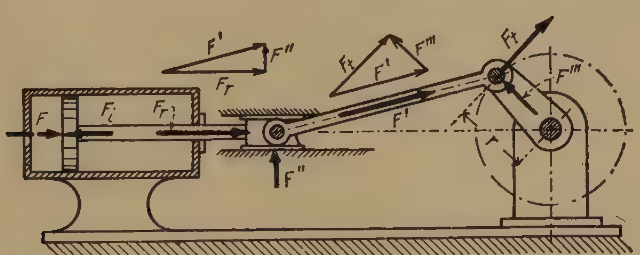


FIG. 198.

F = force due to the working medium;
F_i = inertia force;
$F_r = F - F_i$ = resultant force on crosshead;
F' = connecting rod thrust;
F'' = force exerted by guides on crosshead;
F_t = tangential force at crank pin;
F''' = radial force at crank pin.

varying with the angle between the crank and the connecting rod and with the resultant of the fluid pressure and inertia forces of the reciprocating parts. Figure 198 illustrates this with space and force diagrams. It is apparent that at either dead center the tangential force, F_t, is zero, and that the torque at the crank varies from zero to a maximum and back to

zero again from dead center to dead center. It follows that, if the rotative work of the main shaft is to be delivered at a fairly uniform rate, there is a deficiency of torque at the dead centers and an excess at some other positions of the crank. This is taken care of by the flywheel which stores up kinetic energy when there is an excess of torque and delivers the kinetic energy in the form of rotative work when there is a deficiency of torque at the crank. The flywheel must be made with a moment of inertia sufficiently great to take care of these inequalities without varying an objectionable amount in speed. Since

$$K' = I_z(\omega_1^2 - \omega_2^2) \div 2,$$

the greater is I_z, the less is $(\omega_1^2 - \omega_2^2)$ that is, the speed variation. The equation for kinetic energy may be expressed

$$K' = I_z(\omega_1 - \omega_2)(\omega_1 + \omega_2) \div 2,$$

in which $(\omega_1 - \omega_2)$ is the allowable change in speed, and $(\omega_1 + \omega_2) \div 2$ is the average speed in radians per second.

PROBLEM 306. There is a deficiency of torque at the crank of a steam engine of 5000 lb.-ft. when the crank passes through an angle of 30°; 15° before dead center and 15° after dead center. The flywheel has a moment of inertia of 353 units of mass × ft.² and the average revolutions per minute is 240. What is the variation in speed expressed in per cent above and below the average? *Ans.* 0.585 per cent.

120. Power is the time rate of doing work. Comparing the work effects of two forces, one small and the other large, the velocity of displacement being the same, it is obvious that the smaller force can accomplish an amount of work equal to that delivered by the larger, if more time is allowed the smaller. In order to fix the capacity of a machine, the work delivered per unit of time must be determined. If this were expressed in foot-pounds per second or per minute, the ratings for power would involve inconveniently large numbers. Consequently, a larger unit is employed, namely, the horse-power.

121. One Horse-Power equals 33,000 foot-pounds of work per minute. In order to calculate the horse-power produced or delivered it is only necessary to calculate the work done in a given time, reduce to foot-pounds per minute and divide by 33,000.

In this book the abbreviation "Hp." will be used for horse-power. As in the case of "work" this will be classified as *supplied*, or Hp. input, *utilized* and *lost* horse-power.

122. Efficiency can be expressed, as for work, by

$$E = \frac{\text{Useful Hp.}}{\text{Input Hp.}} = 1 - \frac{\text{Lost Hp.}}{\text{Input Hp.}}.$$

PROBLEM 307. The draw-bar pull from a locomotive drawing a 125-ton train up a 1 per cent grade is 3750 lbs. If the speed of the train is 20 miles per hour, what is the horse-power delivered to the draw-bar? If the speed is 30 miles per hour?
Ans. 200, 300 Hp.

PROBLEM 308. An airplane, travelling at 120 miles per hour at a constant level has a horizontal pull at the propeller shaft of 500 lbs. If the efficiency of propulsion is 33 per cent, what size motor is required? *Ans.* 480 Hp.

PROBLEM 309. A belt and pulley drive delivers 50 Hp. from a 3-ft. diameter pulley turning at 120 r.p.m. The ratio of the belt tension on the tight side to that on the slack side 3 : 1. What is the belt tension on the tight side if there is no slippage? *Ans.* 2190 lbs.

PROBLEM 310. A 24-in. gear is driving a 12-in. gear. If the input horse-power is 20 when the large gear is turning at 100 r.p.m., and the efficiency of transmission is 95 per cent, what is the useful horse-power delivered by the 12-in. gear?

123. Indicated horse-power. This term refers to the horse-power developed in the cylinder of a piston engine, and is derived from the instrument used for measuring the pressure of the working medium, namely, the "engine indicator."

By "pressure" is meant the force exerted on a unit area. In engineering it is usually expressed in pounds per square inch, and will be designated here by the symbol F_u. Fluid pressure acts perpendicularly to any surface confining the fluid. The total force exerted by fluid pressure upon a confining wall equals the product of the pressure and the area of the wall. Calling this area Q,

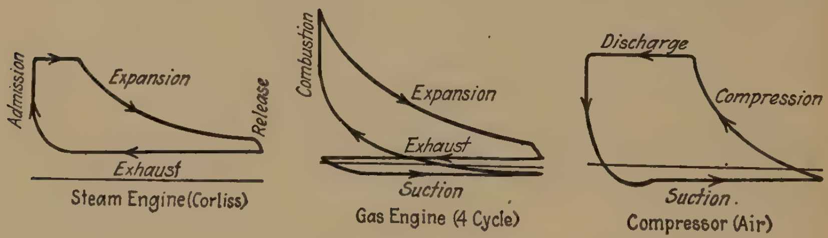
$$\text{Total force} = F_u \times Q,$$

in which, if F_u is taken in pounds per square inch, Q must be in square inches, or if F_u is taken in pounds per square foot, Q must be in square feet.

The engine indicator serves to make an autographic diagram of the pressure of the working fluid at all parts of the cycle. By "cycle" is meant a complete series of events in the cylinder, at the end of which the condition of the working medium is the same as at the start. The "indicator diagram" is a closed curve, any point on which indicates the pressure in the cylinder on one side of the piston, by the height of a point from the base line (atmospheric pressure) at a corresponding position of the piston shown by the horizontal distance of the point from either end of the diagram. The length of the diagram is proportional to the stroke of the piston,

and the right and left hand ends of the diagram represent dead center positions of the piston.

Figure 199 shows indicator diagrams from three types of engines, each representing the variation in pressure on one side of the piston throughout a cycle. For " double-acting " engines, there is a similar cycle for the other



INDICATOR DIAGRAMS

FIG. 199.

side of the piston, 180° out of phase with that on the head end; making two working strokes per revolution of the crank.

The gas-engine, when built on the so-called "four-stroke cycle" (abbreviated " four-cycle ") principle, requires four strokes to complete a series of events, or cycle. In all but the large sizes, these engines are single-acting, no useful work being done on the crank side of the piston, consequently there is only one effective stroke per cylinder in two revolutions.

Referring to the diagrams of Fig. 199, lines drawn to the *right* represent *work done by the working medium on the piston*; lines drawn to the *left* represent *work done by the piston on the working medium*. The effective pressure is therefore the vertical distance between these two lines, and the average effective pressure, or " mean effective pressure " as ordinarily referred to, is the average height of the diagram, in scale units.

Considering a double-acting engine with one cylinder,

Let Ihp. = Indicated horse power,

F_u = mean effective pressure on the head end, pounds per square inch,

F_u' = mean effective pressure on the crank end, pounds per square inch,

L = length of stroke in feet,

Q = area of piston, head end side of piston, square inches,

Q' = area of piston, crank end side of piston, square inches,

N = revolutions of the crank per minute.

The area Q is equal to that of a circle with a diameter equal to the cylinder bore. The area Q' is less than Q by the area of the piston-rod cross-section. In large engines the piston rod, in some designs, runs clear through each end of the cylinder in which case the value Q is the same for each end.

The work done during one revolution of the crank, on the head end of the piston, is $F_u \times Q \times L$. The work done per minute is $F_uQL \times N$, and the indicated horse-power developed on the head end of the piston is

$$(\text{Ihp.})_h = F_uQLN \div 33{,}000,$$

and on the crank end is

$$(\text{Ihp.})_c = F_u'Q'LN \div 33{,}000,$$

and the total Ihp. is

$$\text{Ihp.} = (\text{Ihp.})_h + (\text{Ihp.})_c,$$

or, approximately,

$$\text{Ihp.} = (F_u + F_u')Q''LN \div 33{,}000,$$

in which Q'' is the average of the two areas or $(Q + Q') \div 2$. Let F_u'' stand for $(F_u + F_u') \div 2$ or the average of the mean effective pressures on the two ends. Then

$$\text{Ihp.} = 2 \times F_u''Q''LN \div 33{,}000 \text{ (very nearly)},$$

the factor 2 being introduced to account for two working strokes per revolution.

It was noted before that the piston has a very variable velocity. The average speed of the piston in feet per minute is $2LN$, since the distance it is displaced each revolution is twice the stroke. The last equation for Ihp. after substituting for $2LN$, its value, V, may, therefore, be expressed,

$$\text{Ihp.} = F_u''Q''V \div 33{,}000 \text{ (for double-acting engines only)},$$

in which V is the average piston speed in feet per minute.

PROBLEM 311. A steam engine has a cylinder 10 ins. in diameter, and a piston rod 2 ins. in diameter. The stroke is 16 ins. and the revolutions per minute is 200. The mean effective pressure is 45 lbs. per sq. in. on the head end and 50 lbs. per sq. in. on the crank end. How much work is done on the head end in one revolution? On the crank end? *Ans.* 4710, 5020 ft.-lbs.

PROBLEM 312. In the preceding problem how much work is done on the head end in one minute? On the crank end? *Ans.* 942,000; 1,004,000 ft.-lbs.

PROBLEM 313. What is the Ihp. on the head end? On the crank end? What is the total Ihp. developed? *Ans.* 28.5, 30.4, 58.9 Ihp.

PROBLEM 314. Check the total Ihp. of Problem 313 by the formula using terms for average mean effective pressure and average area.

PROBLEM 315. If the mechanical efficiency of the engine is 90 per cent, how much horse-power is developed at the shaft and how much is lost in friction and windage?

PROBLEM 316. If a single-acting engine has a piston speed of 600 ft. per min., and a mean effective pressure of 60 lbs. per sq. in., with a piston area of 40 sq. in., what is its Ihp.? *Ans.* 21.8 Ihp.

PROBLEM 317. A double-acting steam engine, with a cylinder diameter of 9 ins., and a piston rod 2 ins. in diameter, stroke 24 ins., develops 25 Ihp. when running at 100 r.p.m. What is the mean effective pressure in each end of the cylinder if equal amounts of work are done at each end?

Ans. 32.4, 34.1 lbs. per sq. in.

124. Horse-power of variable speed and multicylinder engines. Automotive engines, with few exceptions, operate on the internal combustion, four-cycle principle, are single acting and have several cylinders. In general the engine indicator cannot be conveniently used to determine mean effective pressures, but the horse-power delivered may be readily measured. If F_u is the average mean effective pressure in all the cylinders, Q the area of the one piston (equal in all of the cylinders), L the length of the stroke in feet, V the piston speed in feet per minute, C the number of cylinders, N the number of revolutions per minute, and E the efficiency,

$$\text{Delivered Hp.} = E \times C \times F_uQL \times (N \div 2) \div 33{,}000.$$

In this term $N \div 2$ is the number of effective strokes per minute per cylinder, and $C \times N \div 2$ is the number of effective strokes per minute in all the cylinders, which will be called N'''. In general, then, the delivered horse-power may be expressed thus:

$$\text{Delivered Hp.} = E \times F_uLQ \times N''' \div 33{,}000.$$

In this, $N''' = CLN \div 2$. Insert for LN, its value $V \div 2$, and $N''' = CV \div 4$. Therefore,

$$\text{Delivered Hp.} = E \times F_uQ \times C \times (V \div 4) \div 33{,}000.$$

PROBLEM 318. The delivered horse-power of automobile engines is often estimated by the formula

$$\text{Hp.} = C(2r)^2 \div 2.5,$$

in which $(2r)$ is the cylinder diameter and C the number of cylinders. This formula is deduced on the assumption that the piston speed is 1000 ft. per min., and the

mechanical efficiency is 85 per cent. What mean effective pressure must there be for an engine to give this rating? *Ans.* 78.9 lbs. per sq. in.

PROBLEM 319. A steel-mill, four-cycle, double-acting gas engine is arranged with two cylinders in tandem and a piston rod running clear through both cylinders. Its cylinder diameter is 47 ins., stroke 5 ft., piston rod diameter 14 ins., and rotative speed 60 r.p.m. If its mechanical efficiency is 87 per cent, and the average mean effective pressure is 70 lbs. per sq. in., what horse-power does it deliver?

Ans. 1750 Hp.

125. Relation between torque and horse-power. The horse-power transmitted in rotation may be expressed thus:

$$\text{Hp.} = 2 \times 3.14 \times \overline{M} \times N \div 33{,}000.$$

From which it follows that the torque necessary to transmit a given horse-power varies inversely with the speed of rotation.

PROBLEM 320. A 75-Hp. steam turbine runs at 12,000 r.p.m. and delivers its power through a 10 to 1 reduction gear to a generator. What is the torque delivered to the generator if the efficiency of the reduction gear is 92 per cent?

Solution. The horse-power delivered to the generator is $75 \times .92 = 69$ Hp. The speed of the generator shaft is $12{,}000 \div 10 = 1200$ r.p.m. Therefore

$$\begin{aligned}\overline{M} &= (33{,}000 \times 69) \div (1200 \times 2 \times 3.14)\\ &= 303 \text{ lb.-ft.}\end{aligned}$$

PROBLEM 321. An automobile of popular make is rated at 22.5 Hp. according to the formula of Problem 318. The stroke is 4 ins. The gear ratio when in high gear is $3\frac{7}{11}$ and when in low gear is 7.51. Taking the efficiency of the transmission gears as 75 per cent, what is the torque at the rear wheels in high and in low gear? If the rear wheels are 30 ins. in diameter, what is the maximum grade this car can climb if its weight with driver is 1800 lbs., when in high gear and in low gear?

Ans. 215, 443 lb.-ft., 9.65 per cent, 20.1 per cent grade.

PROBLEM 322. What car speed corresponds to the data of Problem 321 when in high gear? When in low gear? If the car speed is reduced 10 per cent, the mean effective pressure remaining the same, what is the effect upon the horse-power of the engine and upon the grade that could be climbed with the car in low gear?

Ans. 36.6, 17.8 m.p.h.

126. Brake horse-power, dynamometers. In order to measure the horse-power of an engine or shaft, its energy may be wholly absorbed by a friction brake, the torque on which may be measured, thus enabling a calculation of the power delivered. Because of this method of measurement, the useful power capacity of an engine, that is, the power delivered at the shaft, is referred to as the " Brake horse-power " abbreviated Bhp.

Devices employed for measuring power are called dynamometers. Dynamometers fall into two classes: absorption, with which all the measured energy is absorbed as friction or otherwise; and transmission, in which the measured energy is transmitted through a spring or lever combination by which the torque may be measured. The former class only will be dealt with here.

127. The Prony Brake is the commonest form of absorption dynamometer, and is illustrated by Fig. 200. It consists of a band to which are fastened friction blocks, wrapped around a pulley on the shaft whose power is to be measured, so arranged that by turning a handwheel, S, the friction between the band and the flywheel may be controlled. The band is fastened to an arm, P, and the whole is restrained from rotation by a force measuring device such as a spring scale, as shown, or a platform scale.

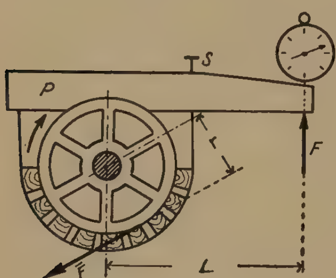


Fig. 200.

In operation the friction from the band blocks on the pulley causes a resisting torque which balances the torque delivered by the shaft. There is friction at each surface of contact; consequently, the resisting torque is composed of a number of forces tangential to the pulley surface, each with a moment arm equal to the radius of the pulley, r. The summation may be represented by a single force, $\overline{F}$, as shown by the figure, whose torque is $\overline{F}r$.

Consider the forces on the brake. If the center of gravity of the brake is directly over the center of the shaft, the weight of the brake has no moment about this center. The forces holding the brake in equilibrium, then, are the pull of the scales, F, acting at arm L and the friction force, $\overline{F}$, acting at arm r. Consequently,

$$FL = \overline{F}r,$$

and the horse-power delivered, or brake horse-power, is

$$\text{Bhp.} = 2 \times 3.14 \times \overline{F}r \times N \div 33{,}000.$$

Inserting for $\overline{F}r$ its equivalent, FL, there is obtained

$$\text{Bhp.} = 2 \times 3.14 \times FL \times N \div 33{,}000.$$

To measure the horse-power, it is not only necessary to determine F, but also the rotative speed, N, in revolutions per minute.

PROBLEM 323. The length of the arm of a Prony brake such as just described is 3 ft. If the brake pulley revolves at 250 r.p.m., and the scales show a pull of 88 lbs., what is the brake horse-power? *Ans.* 12.6 Hp.

PROBLEM 324. Using the data of Problem 323, find the brake horse-power per revolution per minute per pound pull on the scales. *Ans.* 0.00057 Hp.

PROBLEM 325. What should be the length of the arm of a Prony brake so that it will indicate 0.001 Hp. per revolution per minute per pound shown on the scales? *Ans.* 5.26 ft.

PROBLEM 326. With the arrangement shown in Fig. 201, the scales being supported at an angle of 30° from the horizontal, length of brake arm = 5 ft $3\frac{1}{4}$ ins., scale reading = 75 lbs., r.p.m. = 160; what is the brake horse-power? *Ans.* 2.06 Hp.

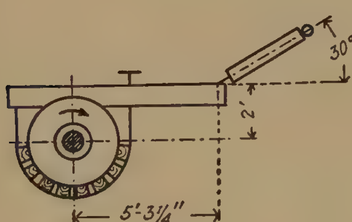


FIG. 201.

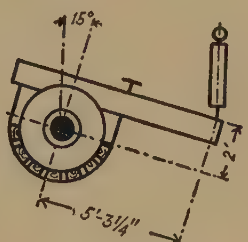


FIG. 202.

PROBLEM 327. Given the same brake as in Problem 326, arranged as in Fig. 202, other data the same, what is the brake horse-power? *Ans.* 12.8 Hp.

128. Unbalanced weight of brake, brake zero. The type of brake just discussed is called balanced, because its center of gravity is directly over the center of rotation, and there is no torque produced by the weight of the brake itself. It is more often the case that the center of gravity of the brake lies somewhere between this position and the point at which the scales are attached. Referring to Fig. 203 it is seen that the weight of the brake, W, will produce two reactions, R and R', as would a simple beam at its two supports. One reaction, R, may be considered as concentrated over the center of the pulley; the other, R' at the scales. The reaction R produces no torque about the center of rotation, and may, therefore, be disregarded. The reaction R' is evidenced by an

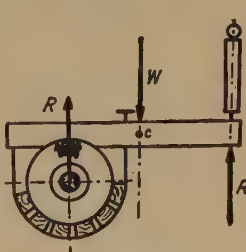


FIG. 203.

additional force indicated by the scales which has nothing to do with the turning effort of the engine. It is customary to determine this reaction experimentally and subtract it from the scale reading in operation, in order to get the net force to be used in the brake horse-power formula.

The reaction R' is called the " unbalanced weight " of the brake, or the " brake zero." It should be noted that it cannot be determined experimentally by reading the scales when the engine is stationary, since, no matter how loosely the band is adjusted, there is always some friction when the brake arm bears on the pulley.

Problem 328. A brake has an unbalanced weight of 10 lbs., and an arm 5 ft. $3\frac{1}{4}$ ins. long. The scales show a total of 70 lbs. when the pulley is turning at 240 r.p.m. What is the brake horse-power?. *Ans.* 14.4 Hp.

Problem 329. Fig. 204 shows a brake set-up in which the pulley turns counter-clockwise. The unbalanced weight is 20 lbs. The scales indicate 50 lbs. at a pulley speed of 120 r.p.m. What is the horse-power developed? *Ans.* 8.43 Hp.

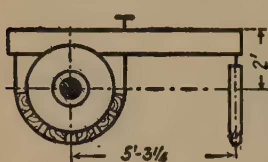


Fig. 204.

Problem 330. A 6-cylinder, 4-cycle standard marine engine, 10-in. bore, 11-in. stroke, when turning at 460 r.p.m. has a mean effective pressure of 80.9 lbs., per sq. in. (Such an engine has three power strokes per revolution.) The propeller has been replaced by a Prony brake and pulley, as in Fig. 204. Unbalanced weight = 30 lbs. If the mechanical efficiency is 90 per cent find the scale reading of the brake at an engine speed of 460 r.p.m. *Ans.* 450 lbs.

PART XIV

MOMENTUM, IMPULSE AND IMPACT

129. Momentum is a conception of Newton, which he called "quantity of motion." Numerically, it is the product of the mass of a body and its velocity. The factors of this quantity are in fundamental units, but no name is assigned to a unit of momentum. It will be referred to in this text as

$$MV = \text{Momentum.}$$

In this treatment, rectilinear momentum only will be dealt with in detail.

130. Impulse is another concept useful in the study of mechanics, and may be defined as the product of a force and the time through which it acts. As with momentum, no name is assigned to unit impulse. When the force is constant,

$$FT = \text{impulse,}$$

the units of F and T being fundamental.

131. Relation between momentum and impulse. From the equation of motion for an unbalanced force; $F = MA$,

$$F = \frac{MV}{T},$$

and

$$FT = MV,$$

that is, the impulse of a resultant force acting upon a body during T seconds produces a momentum, MV, assuming the initial velocity of the body to be zero. If, however, the body has an initial velocity before the force is

applied; from the equation of motion, in which V_1 and V_2 are initial and final velocities, respectively

$$F = \frac{M(V_2 - V_1)}{T},$$

$$FT = MV_2 - MV_1.$$

The physical interpretation of the preceding equation is: *The impulse of a moving force impressed upon a body causes a change of momentum of the body and is equal to that change.*

Since impulse contains the factor F, and momentum, the factor V, both of which are vector quantities; it follows that both impulse and momentum are vector quantities.

If the body referred to is subject to the impulse of several forces, instead of one, as previously considered, then

The change of momentum produced by the impulse of each force is independent of the changes produced by the others, but the total change of momentum equals the vectorial sum of the separate momenta.

Problem 331. A body weighing 58 lbs. is moving with a velocity of 2 ft. per sec. If a 10-lb. force is impressed upon it for 5 sec., what is the change in momentum? *Ans.* 50 units of momentum.

Problem 332. What is the velocity of the body of Problem 331 after 5 sec.? After 10 sec., under the same force of 10 lbs.? *Ans.* 29.8 and 57.6 ft. per sec.

Problem 333. If the body of Problem 331 reaches a final velocity of 80 ft. per sec., how many seconds must the 10-lb. force act? *Ans.* 14 sec.

Problem 334. Two impulses are applied to a body weighing 58 lbs. The first is 12 lbs. and 4 sec. in duration, the second 2 lbs. 24 sec. in duration. The forces are directed at right angles to each other. What is the change in momentum, and what is the direction of the change with reference to the direction of either impulse? *Ans.* 68 units of momentum; 45 deg.

132. Change of momentum under impulse with variable force. If the impressed force, F, changes in magnitude during the period of its action, then

$$\int_0^T F dT = \int_{V_1}^{V_2} M dV = M(V_2 - V_1).$$

This equation may be solved for any unknown if the relation between the force, F, and time is known. This relation is necessary to determine the impulse integral.

***133.* Restatement of Newton's Second Law.** Newton, the founder of the Science of Mechanics, was the first to conceive scientific notions of mass and momentum and to relate these quantities with applied forces. According to his reasoning the second law of motion may be paraphrased briefly as follows:

A resultant force acting upon a body equals the time rate of change of the momentum of the body in the direction of the force; or, in mathematical language, if the initial velocity of the body is zero, the time rate of change of momentum is $MV \div T$, and

$$F = \frac{MV}{T}.$$

As

$$V \div T = A,$$

$$F = MA,$$

as stated in Art. 75 (Newton's Second Law).

***134.* Definition of force from the second law.** Force is an action which when applied to a freely moving body produces a time rate of change of momentum. Quantitatively, a unit force is that which produces a unit change in momentum per unit of time. In English units, and the engineer's unit of mass,

$$1 \text{ lb. force} = \frac{1 \text{ unit of mass} \times 1 \text{ ft. per sec.}}{1 \text{ sec.}}$$

$$= \frac{32.174 \text{ lbs. weight} \times 1 \text{ ft. per sec.}}{32.174 \text{ ft. per sec. per sec} \times 1 \text{ sec.}},$$

or one pound force has a magnitude such as to produce unit momentum per second upon a body of unit mass.

135. Application of the relation $FT = MV$ is made in two ways.

First, it simplifies the solution of problems involving the acceleration of bodies, in which it is required to find time, or the velocity after a given time.

Second, certain principles pertaining to *impact*, or collision of bodies, are derived through laws relating to momentum and impulse.

This article deals with the first application, and the following problems are to be solved only by the relations between impulse and momentum.

PROBLEM 335. If a body weighing 70 lbs. is acted upon by a resultant force of 30 lbs., in what time will its velocity be increased from zero to 50 ft. per sec.?

Solution. The impulse is 30 T, and the change of momentum is $\frac{70}{32.2} \times 50$. Hence,

$$T = \frac{70 \times 50}{32.2} \div 30 = 4.35 \text{ sec.}$$

PROBLEM 336. In what time will a freely falling body of weight W acquire a velocity of 100 ft. per sec. if its initial velocity is zero? *Ans.* 3.1 sec.

PROBLEM 337. An automobile weighing 3000 lbs. has a velocity of 5 miles per hour. Under a constant tractive force of 250 lbs., what will its velocity be after 7 sec., taking the resistance as constant and equal to 20 lbs. per ton?

Ans. 20.9 miles per hr.

PROBLEM 338. A 2500-lb. body receives an impulse during 20 sec. from a variable force, F, the value of which is directly proportional to the time of application, according to the law $F = 1.5\ T$. Starting from rest what is the velocity of the body after one minute?

Solution. The impulse is

$$\int_0^{60} 1.5TdT = \Big[_0^{60} .75T^2 = 2700,$$

and since this equals the change of momentum, $\frac{2500}{32.2} \times V$,

$$V = \frac{32.2}{2500} \times 2700 = 34.8 \text{ ft. per sec.}$$

PROBLEM 339. An automobile weighing 3000 lbs. is accelerated from 25 miles per hr. by a variable tractive force, $F = 200 + 10\ T$. Take the windage and friction resistance as $F' = 20 + .25T^3$. In the equations, the value of F and F' apply simultaneously, at and after a speed of 25 miles per hour, T being zero at 25 miles per hr. In what time will the greatest velocity possible be reached, and what is that velocity? *Ans.* 16 sec., 46.5 miles per hr.

136. Angular impulse and momentum. For a body undergoing rotation, there are similar relations as for rectilinear motion, the difference being that the moment of a force, $M(=FL)$, is used instead of force, moment of inertia instead of mass, and angular velocity instead of linear velocity. Thus, when $\overline{M}$ is constant.

$$\overline{M} \times T = I \times \omega,$$

that is, Angular impulse = Angular momentum.

When $\overline{M}$ varies, and may be expressed as a function of time,

$$\int_0^T \overline{M} dT = \int_{\omega_1}^{\omega_2} I d\omega = I(\omega_2 - \omega_1).$$

PROBLEM 340. Solve Problem 249 for time only, using the relations of this article.

137. Conservation of linear momentum. *If two elastic bodies of mass M and M', approaching each other with velocities V and V' respectively, collide, then the velocity of each body is changed but the sum of the momenta of the bodies is unchanged.*

Mathematically, if the new velocities are V'' and V''',

$$MV + M'V' = MV'' + M'V'''.$$

If the paths of the center of gravity are not in the same right line, and collision occurs, the vector sum of the momenta equals zero.

In the first case, if the bodies have uniform velocities and are assumed to be moving freely without applied force or resistance, then at the instant of contact there is an action from one and a reaction from the other which increases until the maximum compression of the bodies is reached. The only forces involved are mutual and equal actions and reactions, which also operate during the same period of time. The impulses of these forces, therefore, are equal in magnitude, but opposite in direction; hence the sum of the impulses equals zero. In consequence, the change in momentum equals zero; that is, the sum of the momenta is the same before as after collision.

For example, consider two billiard balls, assumed to be perfectly elastic and of the same mass and diameter. If they approach each other in a straight line with the same velocity, then the momentum of one equals that of the other, but they are opposite in sign since, if the velocity of one is plus, the other must be minus. Hence, before collision, the sum of the momenta of the two balls equals zero. After collision they will rebound, the one from the other, with equal and opposite velocities. The sum of their momenta again equals zero; that is, there is no change in the sum of the momenta.

138. Impact may be defined as an impulse of such short time duration as to constitute a collision. The term implies at least two bodies, one of which may be at rest, or both may be in motion.

For simplicity, consider the motion of two spheres of elastic material whose centers of gravity are travelling on the same right line. Collision may occur between these spheres, if they are travelling towards each other from opposite directions, for any value of either velocity. Or collision may occur if the velocities are in the same direction, in which case the velocity of the leading sphere must be less than that of the following sphere.

In the equation of Art. 137,

$$MV + M'V' = MV'' + MV''',$$

take M and M' as the masses of the two spheres under consideration. Let M be the mass of the following sphere when the velocities have the same direction and call V plus. Then V' will be plus, no matter what the direction of V. When the spheres approach each other from opposite directions, call the velocity V plus, no matter what its direction; then V' will be minus.

The velocities, V, V', V'', and V''' are *absolute velocities*, that is, they are referred to the earth as at rest. *The relative velocities* of the two spheres with respect to each other is the *algebraic* difference of their absolute velocities.

Before collision, no matter in which way the spheres approach each other, the

$$\text{relative velocity} = V - V',$$

which yields a positive sign in either approach, since V is greater than V' if the velocities are in the same direction; or, if in opposite directions, V being always called plus, V' is negative and $V - (-V')$ is plus.

At the instant beginning collision, each sphere reacts on the other and a small force exists which grows larger as the material of each sphere is compressed by the impact. This continues until a maximum compression of the spheres on the area of collision is reached. At that instant the absolute velocity of the spheres is the same and the relative velocity zero. Immediately after, the elasticity of the material exerts an action and reaction which separate the spheres. At the instant of separation of contact, the absolute velocities are V'' and V''' and the

$$\text{relative velocity} = -(V'' - V'''),$$

$$= (V''' - V'').$$

The minus sign is necessary to indicate that *after* the collision the distance between the spheres is *increasing;* whereas *before* collision the distance was decreasing.

139. **Coefficient of restitution.** The effects described under Art. 138, from the time of first contact to that of maximum compression, occupy what is called the *period of compression.* The following effects, from the time of maximum compression to the time of separation of the spheres with final absolute velocities of V'' and V''', occupy what is called the *period of restitution.*

If the spheres are perfectly elastic, that is, they regain their original shape after compression, and if there are no other resistances to motion than that due to the impact, then the relative velocities before and after collision would be the same, and

$$V - V' = V''' - V'',$$

from which
$$1 = \frac{V''' - V''}{V - V'},$$

that is, the ratio of the relative velocity after collision to that before is unity.

Solid materials are not perfectly elastic. Consequently there is a loss in relative velocity, which may be expressed by the ratio of final to initial relative velocity thus,

$$k = \frac{V''' - V''}{V - V'},$$

in which k is called the *coefficient of restitution* and is always less than unity.

By "perfectly elastic" is meant in this context, not only the property of returning to original shape after deformation under an increasing force, but also that the *rate* of resuming original shape with respect to the applied force is the same during the period of restitution as during the period of compression. This is a theoretical impossibility with any engineering material since it requires that the work done during restitution equals that of compression. This requirement is equivalent to that of a machine having 100 per cent efficiency.

With actual materials k may approach unity very nearly, but the exact value of k is always a matter of uncertainty, especially with dissimilar materials.

***140.* Problems involving impact and momentum.** The equation for the conservation of linear momentum may be expressed thus

$$WV + W'V' = WV'' + W'V''',$$

since in each expression for momentum, mass equals the corresponding weight divided by 32.174 which number may be cancelled from the equation. The new equation is expressed in weights instead of masses, and velocities the same as before.

The use of this equation together with a knowledge of k in the following

$$k = \frac{V''' - V''}{V - V'},$$

and the necessary initial data enable the solution of problems.

PROBLEM 341. A body, P, weighing 12 lbs. and moving with a velocity of 16 ft. per sec. collides with another body, Q, weighing 24 lbs. and travelling at 8 ft. per sec. Both bodies are moving to the right. The coefficient of restitution is .75. What are the relative velocities before and after impact, and what are the absolute velocities after impact?

Solution. The following body is P. By convention, its absolute velocity is plus and all absolute velocities to the right are plus, since P moves to the right.

Relative velocity before impact $= 16 - 8 = 8$ ft. per sec. Substituting in

$$k = \frac{V''' - V''}{V - V'}.$$

Gives

$$.75 = \frac{V''' - V''}{8},$$

and the relative velocity after impact is $V''' - V'' = 6$ ft. per sec.

Also

$$V''' = V'' + 6.$$

Substituting all knowns in the equation between momenta

$$12 \times 16 + 24 \times 8 = 12V'' + 24V''',$$

$$= 12V'' + 24\,(V'' + 6),$$

from which the absolute velocity of P after impact,

$$V'' = +\,6.67 \text{ ft. per sec.}$$

Substituting in $V''' = V'' + 6$ gives the absolute velocity of Q.

$$V''' = +\,12.67 \text{ ft. per sec.},$$

both of which are to the right as denoted by their signs.

PROBLEM 342. In the preceding problem, it may be noted that the momentum of P is equal to the momentum of Q. Keeping the same numerical data for momenta, suppose the direction of Q be reversed so that collision occurs by approach from opposite directions. Since the momenta of the two bodies are equal in magnitude, it is to be expected that, after collision, they will rebound with equal and opposite velocities when $k = 1$. By the method of Problem 341, prove this to be true, and find the velocity in amount and direction after impact.

Ans. $V'' = -12$, $V''' = +12$ ft. per sec.

PROBLEM 343. When a billiard ball, Q, is at rest on a table, if it is struck by another ball, P, of the same mass, with velocity V before impact; prove that, after impact, the velocity of P is zero, and of Q is V.

The preceding, from the beginning of Art. 138, has to do with only one type of impact, called *direct central impact;* that is, the center of gravities of the impacted bodies have relative velocities on the same right line. Many other examples and problems of this type of impact could be presented, as well as of other types of impact. These are not included here, mainly for the reason that the engineer rarely encounters problems involving impact and momentum relations, although the effect of impact upon internal stresses of materials may be of considerable engineering importance. This, however, is a subject falling under the *Mechanics of Materials.* In any event the uncertainty of the coefficient of restitution, especially when the impacting bodies are of different materials, makes the calculations given in this article of questionable value.

PART XV

THE CONSISTENCE AND CONVERSION OF COMPOUND ENGINEERING UNITS

141. The student of any mathematical science should keep in mind two things when applying the laws, expressed as equations, to the solution of problems in that science.

First, it is necessary to interpret the physical meaning of each equation used; to look upon it not as a formula, but an abbreviation, in the language of mathematics, to represent a physical condition or law. If formulas are depended upon without this interpretation, the student becomes merely an operator of a device in which he inserts known quantities and grinds out unknowns. He is unable to reconstruct or to create, and stands in the same position with relation to science, or engineering, as the phonograph does to the composer of music.

Second, when equations representing physical phenomena are deduced or applied to problems, the units of each expression in any such equation must be the same as the units of all other such expressions in that equation. As a homely illustration, it is untrue to say, in a mathematical sense, that x apples $+ \, y$ pears $= z$ oranges. The units are not consistent. But it may be correctly stated if x, y, and z are the cost per apple, per pear and per orange, and if n, n' and n'' are the number bought, respectively, of each,

$$xn + yn' = zn''.$$

Each expression now represents the total cost of the item designated, and if one expression is in cents, the others must be in cents; or if one is in dollars, the other must be in dollars.

An examination of the expressions of an equation reveals not only the units to be employed, but also promotes its physical interpretation.

142. Analysis of compound units in mechanical equations. Article 10 states that the fundamental units in mechanics in this book are force,

length and time, designated by F, L, and T; and that mass, M, is then a derived unit. Any compound quantity in mechanics may be reduced to the terms, F, L, and T, or two of them or to a pure number.

Area, being the product of two linear dimensions, is written L^2.

Volume, being the product of three linear dimensions, is written L^3.

Consider the units of pressure, F_u. This is defined as the total force upon an area divided by the number of units of the area; or force per unit area. Then

$$\text{Units of } F_u = \frac{F}{L^2},$$

which is preferably written

$$FL^{-2}.$$

Consider the units of the quantity π which equals

$$\frac{\text{Circumference of a circle}}{\text{Diameter of that circle}}.$$

Both numerator and denominator are in linear dimensions and

$$\text{Units of } \pi \text{ are} \qquad \frac{L}{L} = \text{a pure number.}$$

That is, π has no units, and therefore has the same value, no matter if the circumference and diameter are simultaneously chosen in inches, feet, centimeters or meters.

When investigating the units of a compound quantity, they may be cancelled in numerator and denominator according to the rules of algebra, in order to disclose the basic fundamental units.

For example, take the quantity called the "modulus of elasticity." This is defined as the number of pounds per square inch necessary to apply to a material in tension in order to increase 1 inch of its length to 2 inches (assuming that the elongation is proportional to the applied force); or in other words, it is the tensile force applied to a material of unit cross-section area, per unit of elongation per unit of length. The fundamental units are written

$$\frac{F}{L^2} \div \frac{L}{L}.$$

The units of the divisor here cancel, therefore the expression reduces to

Units of F_u are $\qquad FL^{-2}$,

in which L is in inches.

If force is taken as a fundamental unit, as in this book, then from $F = MA$, and $M = F \div A$, or

$$\text{Units of } M \text{ are} \qquad \frac{F}{LT^{-2}} = FL^{-1}T^2.$$

If mass is taken as a fundamental unit, as is done in many cases, then the units of force are those of MA, or

Units of F are $\qquad MLT^{-2}$.

Consider now, an equation such as

$$\overline{W} = MV^2 \div 2,$$

that is, the work done by a moving force upon a free body equals its gain in kinetic energy. The units on the one side of the equation must be the same as on the other.

The units of $\overline{W}$ are $\qquad LF$.

Using the units of mass as given above, and the units $V^2 = L^2T^{-2}$.

The units of $MV^2 \div 2$ are $\quad FL^{-1}T^2 \times L^2T^{-2} = FL$,

which discloses that kinetic energy is in foot-pounds and that V must be taken in feet per second, in order that the two sides of the equation be consistent in units.

Suppose it is required to determine whether θ is in degrees or radians in the following equation

$$\frac{F}{F'} = e^{f\theta},$$

which applies to the ratio of tensions of a belt running over a pulley. In this, F is the tension on the tight side of the belt; F', the tension on the

slack side; e is the base of the Napierian system of logarithms; f the coefficient of friction between belt and pulley, and θ the angle of contact of the belt with the pulley.

The left-hand side of the equation is a ratio of forces and therefore a pure number.

The equation may be expressed

$$\log_e (F \div F') = f\theta,$$

the left side of the equation remains a pure number; therefore the right side must be. f is a pure number since it is a ratio of forces; therefore θ must be. Consequently θ is in radians and not degrees.

143. **The conversion of compound engineering units.** It is often desirable to convert an equation or coefficient in mechanics from English to metric values, or vice versa. It is then necessary to determine the fundamental units of the compound quantity. Using the gravitational units in both systems, time being measured in both by seconds or minutes, it is only necessary in both systems to employ the following equivalents:

L	1 centimeter = 0.3937 inch	1 inch = 2.45 centimeters.
F	1 kilogram = 2.205 pounds	1 pound = 0.4536 kilogram.

As an example, suppose that a pressure in kilograms per square centimeter is indicated by a metric pressure gage, and it is desired to find the conversion factor to change the metric readings to pounds per square inch.

The fundamental units of this quantity are FL^{-2} in which F is in kilograms and L in centimeters, in the metric system. Multiply F by its equivalent 2.205 in pounds; and L by its equivalent 0.3937 in inches, the conversion factor is

$$\frac{2.205}{0.3937^2} = 14.2,$$

that is, 14.2 lbs. per sq. in. = 1 kg. per sq. cm.

Referring again to the modulus of elasticity whose units have been shown to be FL^{-2} or the same as unit pressure, let us find the English equivalent of the metric value of this quantity; equal to 2,110,000 kg. per sq. cm. per cm. per cm.

Multiply this value by the conversion factor, 14.2 and

$$14.2 \times 2{,}110{,}000 = 30{,}000{,}000 \text{ lbs. per sq. in. per in. per in.}$$

It is in a similar manner that the constants of empirical formulas may be converted. For example, Francis' formula for suppressed weirs is, in English units,

$$Q = 3.33\, L'\sqrt{L^3},$$

in which Q is the discharge in cubic feet per second; L', the breadth of, and L the head on, the weir in feet. The metric coefficient may be ascertained thus:

$$\text{Units of 3.33 are} \quad \frac{Q}{TL'L^{3/2}} = L^3T^{-1}L^{-1}L^{-3/2} = T^{-1}L^{1/2}.$$

Hence, by multiplying 3.33 by $\sqrt{12 \times 0.0254}$, the metric value, 1.84, of this coefficient is obtained, giving Q in cubic meters per second.

ADDITIONAL PROBLEMS

PART I

The first number of each Problem number indicates the part of text appropriate to it. The second number is the sequence number for that part.

PROBLEM 1–1. State mathematically the fundamental units of the relations:

(a) One cu. in. of gold weighs 0.72 lb.
(b) Pressure in pounds per square inch.
(c) Weight in pounds per gallon.
(d) Thickness of covering in inches per square foot.
(e) A horse-power equals 550 foot-pounds per second.

PROBLEM 1–2. Do the following forces fall under the definition of Art. 2:

(a) The expansion of water when freezing.
(b) The expansion of a steel rail on a railroad in the heat of summer.
(c) The turning of a ship's propeller in water.
(d) The action of the earth's magnetic pole on a compass needle.
(e) The action of a balloon filled with hydrogen when anchored to the earth?

PROBLEM 1–3. Carbon monoxide is classified as an odorless, tasteless and colorless gas. Is it "matter" as defined under Art. 3? Is electricity "matter"?

PROBLEM 1–4. A body weighing 100 lbs. is suspended by the system of cords shown in Fig. 1–1 so arranged that the spreader member, *tu* (weight to be neglected) and the cord, *pq*, are horizontal. Draw the space diagrams for the following:

(a) For the joint *s*.
(b) For the joint *t*.
(c) For the joint *p*.
(d) For the cord *us*.

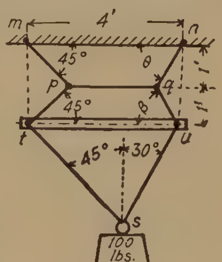


FIG. 1–1.

PROBLEM 1–5. If in Fig. 1–1 the angle θ = angle β = 60° when the weightless spreader *tu* and cord *pq* are horizontal draw the space diagram for the following:

(a) For the joint *q*
(b) For the member *tu*
(c) For the cord *mp*.
(d) For the cord *nq*.

PROBLEM 1–6. If the angles of Fig. 1–1 are as given in the preceding problem draw the space diagram showing the reactions.

PROBLEM 1–7. A rigid metal sphere weighing 50 lbs. is resting upon the center of a flat, rigid table. It is acted upon by a vertical force of 10 lbs. acting downward upon a vertical diameter. Draw the space diagram for the sphere.

PROBLEM 1-8. A man is hanging from a pair of swinging rings by clasping one ring in each hand. His body is so suspended that each arm carries half of his weight. The rings are of such a distance apart that his arms are vertical. Draw the space diagram for

(*a*) The rings and man, showing reactions.
(*b*) The swinging rings.
(*c*) The man.

PROBLEM 1-9. A slack wire expert weighing 125 lbs. stands 5 ft. from one end of a 20-ft. slack wire. He depresses the wire 2.5 ft. Draw the space diagram for the wire, assigning directions to all forces.

PROBLEM 1-10. The ladder shown in Fig. 1, page 5 is hinged to the wall at the top. Draw the space diagram for the ladder, assuming no friction to exist at the floor; assign directions to all forces.

PROBLEM 1-11. A body of inconsiderable weight is pressed against a frictionless vertical wall by an horizontal force, F. Can equilibrium exist? Why? If the force, F, is inclined slightly from the horizontal?

PROBLEM 1-12. Can equilibrium exist for the ladder of Fig. 1, page 5 if both wall and floor are frictionless, other conditions remaining the same? Can equilibrium exist in the ladder of Fig. 8, page 9, if the wall is frictionless?

PROBLEM 1-13. What is the moment of the tension in the cord, mp, of Fig. 1-1 about the point s? About n? The tension in the cord mp may be taken as 51.5 lbs.

PROBLEM 1-14. What is the moment of the tension in the cord, ts, of Fig. 1-1 about u? About p? About t? The tension in cord ts may be taken as 51.5 lbs.

PROBLEM 1-15. If the compression in the member, mn, of Fig. 12, page 10, be taken as 1000 lbs. and the tension in the cable, pn, as 1414 lbs. when the weight, W, is 1000 lbs., the forces in mn and pn being co-linear with the respective members, what are the moments of the three forces about a point in the wall midway between m and p?

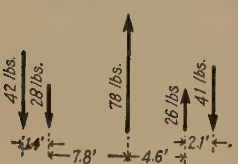


FIG. 1-2.

PROBLEM 1-16. What is the moment of the gravity force of Fig. 1-1 about (*a*) a point on pq extended, and located 5.58 ft. to the right of q, and (*b*) a point on tu extended, and located 5 ft. to the right of u?

PROBLEM 1-17. What are the moments of the forces of Fig. 1-2 about a point on the 42-lb. force?

PROBLEM 1-18. In Fig. 12, page 10, what is the moment of the force, W, about the point, n? Why?

PART II

In Problems 2-1 to 2-8 inclusive locate the resultant in magnitude, location and direction. All forces are co-planar.

PROBLEM 2-1. Two 20-lb. forces acting to the right and 7 ft. apart.

Ans. 40 lbs. to the right midway between the forces.

PROBLEM 2–2. F = 20 lbs. to the right, F' = 40 lbs. to the right 6 ft. below F.
Ans. 60 lbs. to right 4 ft. below F.

PROBLEM 2–3. F = 25 lbs. downward, F' = 55 lbs. upward 8 ft. to the right of F.
Ans. 30 lbs. upward 14.76 from F and 6.67 ft. from F'.

PROBLEM 2–4. F = 28 lbs. downward, F' = 48 lbs. downward 6 ft. to the right of F, F'' = 24 lbs. upward 6 ft. to the right of F'.

PROBLEM 2–5. F = 10 lbs. downward, F' = 40 lbs. downward 4 ft. to the right of F, F'' = 80 lbs. downward 7 ft. to the right of F', F''' = 2 lbs. downward 6 ft. to the right of F''.
Ans. 132 lbs. downward 8.14 ft. to the right of F.

PROBLEM 2–6. The forces shown in Fig. 1–2 with respect to the 28-lb. force?
Ans. 7 lbs. downward 56.5 ft. to the left.

PROBLEM 2–7. Let 100 lbs. be added to each of the forces of Fig. 1–2. (The 42-lb. force becomes 142 lbs. the 78-lb. force 178 lbs. etc.)
Ans. 107 lbs. downward 10.28 ft. to the left of the 128-lb. force.

PROBLEM 2–8. Consider the set of forces of Fig. 1–2 to be rotated through 180° about an axis at the left side perpendicular to the plane of the forces so that the forces are not only reversed in direction but in location. (The 41-lb. force downward now located at the right end of the set becomes a 41-lb. force upward at the left end.) What is the location and magnitude of the resultant with respect to the 28-lb. force?

PROBLEM 2–9. What is the equilibrant of two 40-lb. forces acting upward and 7 ft. apart.
Ans. 80 lbs. downward midway between the forces.

PROBLEM 2–10. F = 60 lbs. to the left, F' = 70 lbs. to the left 4 ft. below F, F'' = 80 lbs. to the right 2 ft. below F'. What is the equilibrant?
Ans. 50 lbs. to the right 4 ft. above F.

PROBLEM 2–11. What is the equilibrant of the forces of Prob. 2–7?

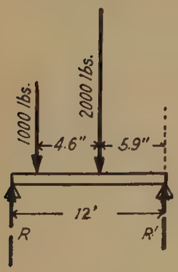


FIG. 2–1.

PROBLEM 2–12. What are the reactions of a simple beam loaded and supported as shown in Fig. 2–1?
Ans. 1860, 1140 lbs.

PROBLEM 2–13. What are the reactions of the simple beam of Fig. 2–1 if the loads are interchanged? *Ans.* 2240, 760 lbs.

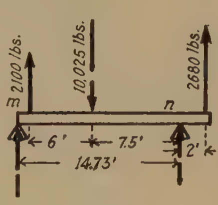


FIG. 2–2.

PROBLEM 2–14. Assume a beam loaded and supported as in Fig. 2–2. What are the reactions?
Ans. 3570, 1690 lbs.

PROBLEM 2–15. Considering that the two end loads of Fig. 2–2 are interchanged what then are the reactions?
Ans. 2945, 2300 lbs.

PROBLEM 2–16. Two men are carrying a 300-lb. weight on a slender rod (weight neglected) 17.2 ft. in length between hand holds. It is desired that one man carry three times the weight carried by the other. Where should the weight be hung?
Ans. 4.3 ft. from man carrying heavier load.

PROBLEM 2-17. What are the magnitudes and directions of the reactions of the beam loaded as shown in Fig. 2-2 if the support at n is moved 13.5 ft. to the left of its present position?

PROBLEM 2-18. What are the magnitudes and directions of the reactions of a beam loaded as shown in Fig. 2-2 if the support now at m is moved to a position at the right end of the beam?

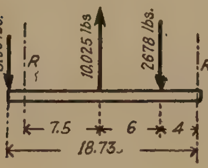


FIG. 2-3.

PROBLEM 2-19. A beam loaded as shown in Fig. 2-3 has its reactions along the dotted lines shown. What are their magnitudes and directions?

Ans. 1802, 2445 lbs., both downward.

PROBLEM 2-20. Consider the loads of the beam of Fig. 2-3 to be reversed in directions. What then are the magnitudes and direction of the reactions?

PROBLEM 2-21. The beam of Fig. 2-1 has, beside the concentrated loads shown, a uniform load of 273 lbs. per foot of length. What are the reactions?

PROBLEM 2-22. The beam of Fig. 2-2 has, beside the concentrated loads shown, a uniform load of 155 lbs. per foot of length. What are the reactions?

PROBLEM 2-23. The beam of Fig. 2-3 has, beside the concentrated loads shown, a uniform load of 105 lbs. per foot of length. What are the magnitudes and directions of the reactions if their lines of action are along the dotted lines?

PROBLEM 2-24. The beam of Fig. 2-3 has, beside the concentrated loads shown, a uniform load of 170 lbs. per foot over the left half of its length and a uniform load of 205 lbs. per foot over its right half. What then are the magnitudes and directions of the reactions if their lines of action are along the dotted lines?

PROBLEM 2-25. The balance beam of Fig. 2-4 is to be held in equilibrium by a vertical force, F, at n. What is the magnitude of this force?

Ans. 12 lbs.

PROBLEM 2-26. The balance beam of Fig. 2-4 is to be held in equilibrium by a force at n which makes an angle of 30° with the beam. What is the magnitude of this force?

Ans. 24 lbs.

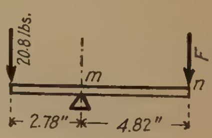


FIG. 24.

PROBLEM 2-27. What is the resultant of a clockwise couple formed of two 50-lb. forces 6 ft. apart, a clockwise couple formed of two 100-lb. forces 3 ft. apart and a counterclockwise couple formed of two 80-lb. forces 3 ft. apart?

PROBLEM 2-28. What is the resultant of a clockwise couple formed of two 73-lb. forces 8.1 ft. apart and a counterclockwise couple formed of two 65.7-lb. forces 9 ft. apart?

PROBLEM 2-29. What is the resultant of the couples given in Problem 2-27 with a horizontal force F of 23 lbs. to the right?

Ans. 23-lb. force to the right 15.6 ft. above F.

PROBLEM 2-30. What is the resultant of the couples of Prob. 2-28 with two horizontal forces of 10 lbs. each, both acting to the right and 2 ft. apart?

Ans. 20 lbs. to right midway between the forces.

PROBLEM 2–31. What is the resultant of the couples of Prob. 2–27 with two horizontal forces of 10 lbs. each both acting to the right and 2 ft. apart?

Ans. 20 lbs. to right 17 ft. above upper force.

PROBLEM 2–32. What is the resultant of a clockwise couple formed of two 60-lb. forces 6 ft. apart, a counterclockwise couple formed of two 50-lb. forces 60 ins. apart, and a counterclockwise couple formed of two 160-oz. forces 5 ft. apart?

Ans. 60 lb.-ft. clockwise couple.

PROBLEM 2–33. What is the resultant, with respect to the force, F, of the forces shown in Fig. 2–5? *Ans.* 20 lb. to left 7 ft. above F.

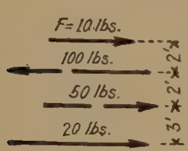


FIG. 2–5.

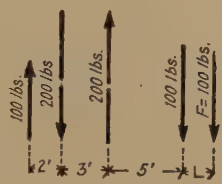


FIG. 2–6.

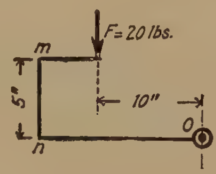


FIG. 2–7.

PROBLEM 2–34. What is the equilibrant of the forces shown in Fig. 2–6?

Ans. 100 lb. downward 4 ft. to right of F.

PROBLEM 2–35. Resolve the force, F, shown in Fig. 2–7 into a force through the point, o, and a couple through m and n.

Ans. At o, 20 lbs. down; counterclockwise couple of 40-lb. forces.

PART III

PROBLEM 3–1. Locate the center of gravity, with respect to the point, o, of the homogeneous solid (which may be considered as being formed of two prisms P and Q) with the dimensions shown in Fig. 3–1.

Ans. 5, 2, −2 ins.

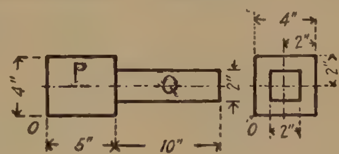


FIG. 3–1.

PROBLEM 3–2. Locate, with respect to the point, o, the center of gravity of the solid shown in Fig. 3–1 if the prism P is formed of material of twice the density of that of Q. *Ans.* 4, 2, −2 ins.

PROBLEM 3–3. Locate the center of gravity with respect to the point o of the solid of Fig. 3–1 if the prism P is formed of material having half the density of that of Q.

Ans. 6.25, 2, −2 ins.

PROBLEM 3–4. Locate the center of gravity with respect to the point o of the 6-in. slotted cube shown in Fig. 3–2.

Ans. 2.72, 3, −3.29 ins.

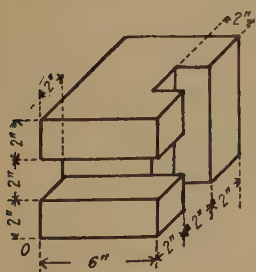


FIG. 3–2.

PROBLEM 3–5. Locate the center of gravity with respect to the center of the sphere of the iron casting shown in Fig. 3–3. This casting may be considered as being formed of a sphere, a cylindrical rod and a cylinder, all concentric about a longitudinal axis. *Ans.* 9.6, 0, 0 ins.

PROBLEM 3–6. Locate the centroid, with respect to *o*, of the area shown in Fig. 3–4. *Ans.* 0.81, −1.82 ins.

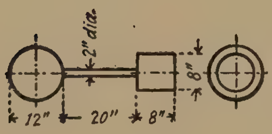


FIG. 3–3.

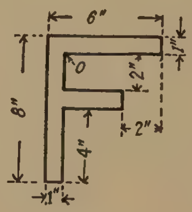


FIG. 3–4.

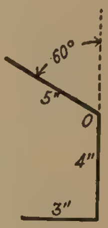


FIG. 3–5.

PROBLEM 3–7. Locate the centroid, with respect to *o*, of the bent wire shown in Fig. 3–5. *Ans.* −1.28, −1.15 ins.

PROBLEM 3–8. Locate the centroid, with respect to *o*, of the shaded area of Fig. 3–6. This area is symmetrical about a vertical axis. *Ans.* 1.5, 2.67 ins.

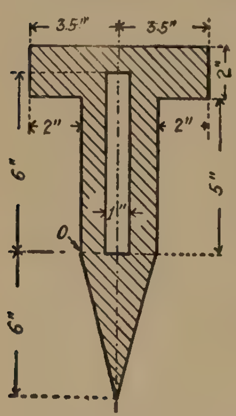


FIG. 3–6.

PROBLEM 3–9. What is the volume of the solid of revolution formed when the area of Fig. 3–4 is revolved about an axis 2 ins. distant from and parallel to the 8-in. side? *Ans.* 383.5 cu. ins.

PROBLEM 3–10. What is the volume of the solid of revolution formed when the area of 3–4 is revolved about an axis 1 in. above and parallel to the 6-in. side? *Ans.* 384 cu. ins.

PROBLEM 3–11. What is the volume of the solid of revolution formed when the area of Fig. 3–6 is revolved about an axis 2 ins. above and parallel to the 7-in.side? *Ans.* 1271 cu. ins.

PROBLEM 3–12. What area of revolution is formed when the bent wire of Fig. 3–5 is revolved about an axis parallel to and 3 ins. below the 3-in. side? *Ans.* 441 sq. ins.

PROBLEM 3–13. What is the volume of revolution formed when a triangle with a base of 6 ins. and an altitude of 3 ins. is revolved about an axis parallel to and 7 ins. below the base? *Ans.* 454 sq. ins.

PART IV

Solve the following three problems by the "Parallelogram of Forces."

PROBLEM 4–1. If in Fig. 54, page 27, F is 60 lbs., F' is 40 lbs., and β is 30°, what is F_r in magnitude and direction?

PROBLEM 4-2. If in Fig. 54, F is 100 lbs., F' is 75 lbs. and β is 120°, what is F_r in magnitude and direction?

PROBLEM 4–3. If the forces of Fig. 56, page 28, are doubled in magnitude what then is the equilibrant in magnitude and direction?

PROBLEM 4–4. Check the solutions of Problems 4–1 and 4–2 by the method of the algebraic solution of the triangle of forces.

PROBLEM 4–5. A force of 1000 lbs. is inclined 30° from the horizontal. What are its vertical and horizontal components? *Ans.* 500, 866 lbs.

PROBLEM 4–6. A force of 1785 lbs. is inclined 45° from the vertical. What are its vertical and horizontal components? *Ans.* 1262, 1262 lbs.

PROBLEM 4-7. One guy wire of a derrick is subjected to a tension of 50,000 lbs. If it makes an angle with the ground whose tangent is 0.75 what are its vertical and horizontal components? *Ans.* 30,000; 40,000 lbs.

PROBLEM 4–8. If the oblique components of a horizontal force of 1000 lbs. are to make an angle of 60° with each other and one of them is to be at right angles to the force what are their magnitudes? *Ans.* 1155, 577.5 lbs.

PROBLEM 4–9. If the oblique components of a horizontal force of 1000 lbs. are to make an angle of 30° with each other and one is to be at right angles with the force what are their magnitudes? *Ans.* 2000, 1732 lbs.

PROBLEM 4–10. If the oblique components of a horizontal force of 1000 lbs. are to make an angle of 60° with each other and with the force what are their magnitudes? *Ans.* 1000, 1000 lbs.

PROBLEM 4–11. Draw the space and force diagrams for the drum of Fig. 4–1 if the force, F, is horizontal and is of just sufficient magnitude to raise the drum (weight, W) over the obstruction. Mark angles and forces plainly. The line of action of F passes through m the uppermost point of the drum.

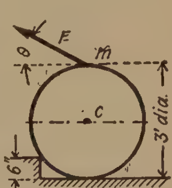


FIG. 4–1.

PROBLEM 4–12. Same as the preceding problem but the force F is at an angle of 30° with the horizontal.

PROBLEM 4–13. Same as Prob. 4–11 but the force F is at an angle of 60° with the horizontal.

PROBLEM 4–14. Same as Prob. 4–11 but the force F is a horizontal force applied at the center of gravity, c.

PROBLEM 4–15. Draw the space and force diagrams for the pin-connected boom of Fig. 4–2 if a load of 1000 lbs. is suspended:

(a) From the point *t*.
(b) From the point *u*.
(c) From the point *s*.

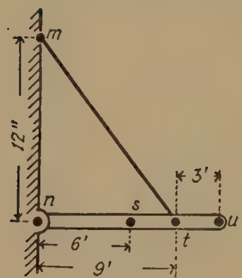


FIG. 4–2.

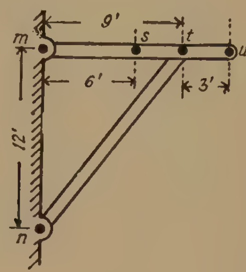


FIG. 4–3.

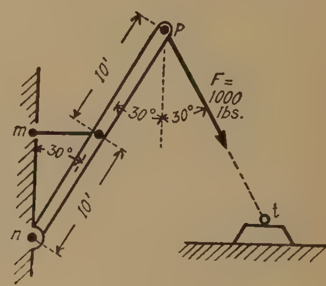


FIG. 4–4.

PROBLEM 4–16. Draw the space and force diagrams for the pin-connected structure of Fig. 4–3 if a load of 1000 lbs. is suspended:

(a) From the point *t*.
(b) From the point *u*.
(c) From the point *s*.

PROBLEM 4–17. Draw the space and force diagrams for the pin-connected structure of Fig. 4–4 showing the reactions at *m* and *n*.

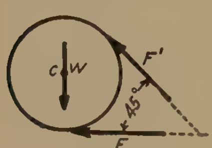


FIG. 4–5.

PROBLEM 4–18. Draw the space and force diagrams for the boom, *np*, of Fig. 4–4 showing the forces on it.

PROBLEM 4–19. Fig. 4–5 represents a wheel of a weight, *W*, in space and acted upon by the forces *F* and *F'* as shown. These forces are coplanar with *W*. Is the wheel in equilibrium? Why?

PROBLEM 4–20. In the pin-connected structure of Fig. 89, page 40, the length of the boom, *mq*, is 6 ft. and *pq* is 3 ft. Draw the space and force diagrams for this structure if a load of 1000 lbs. is suspended from *q*.

PROBLEM 4–21. In the pin-connected structure of Fig. 89 the length of the boom *mq* is 4.5 ft. and *pq* is 1.5 ft. Draw the space and force diagrams for this structure if a load of 1000 lbs. is suspended from *q*.

PROBLEM 4–22. Draw the space and force diagrams for the door shown in Fig. 10, page 9, assigning a weight, *W*, and any desired dimensions.

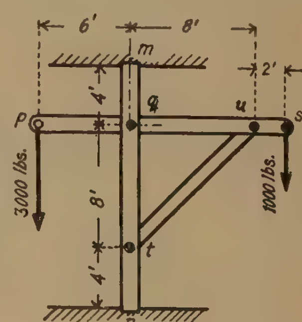


FIG. 4–6.

PROBLEM 4–23. The pin-connected structure of Fig. 4–6 has a counter weight of 3000 lbs. suspended at *p* and a load of 1000 lbs. suspended at *s*. Draw the space and force diagrams showing the reactions at *m* and *n*.

PROBLEM 4–24. What is the force, F, necessary to raise the drum of Fig. 4–1 over the obstruction if the weight of the drum is 100 lbs. and the force, F, is horizontal? *Ans.* 44.7 lbs.

PROBLEM 4–25. Same as Problem 4–24 excepting that θ is 30°?

PROBLEM 4–26. Same as Problem 4–25 excepting that θ is 60°?

PROBLEM 4–27. What is the reason for the variation in the magnitude of the force, F, of Problems 4–24, 4–25 and 4–26?

PROBLEM 4–28. Same as Problem 4–24 excepting that the force, F, is a horizontal force applied at the center of gravity of the drum? *Ans.* 112 lbs.

In Problem 4–29 to 4–32 inclusive solve for the reactions at m and n by the method of "similar triangles." All structures are pin-connected and weights of the members themselves are to be neglected.

PROBLEM 4–29. Fig. 4–2 with a load of 1000 lbs. suspended at t.
Ans. 1250, 750 lbs.

PROBLEM 4–30. Fig. 4–2 with a load of 1000 lbs. suspended at u.
Ans. 1667, 1050 lbs.

PROBLEM 4–31. Fig. 4–2 with a load of 1000 lbs. suspended at s.

PROBLEM 4–32. Assume that the weight of the door shown in Fig. 10, page 9, is 85 lbs. and that the center of gravity, c, is 1.4 ft. from the vertical line of the hinge pins which are 7.2 ft. apart when measured vertically.
Ans. 86.5, 16.5 lbs.

In Problems 4–33 to 4–37 inclusive solve for the reactions at m and n by the method of "components." All structures are pin-connected and weights of members are to be neglected.

PROBLEM 4–33. Fig. 4–3 with a load of 1000 lbs. suspended at t.
Ans. 750, 1250 lbs.

PROBLEM 4–34. Fig. 4–3 with a load of 1000 lbs. suspended at u.
Ans. 1050, 1667 lbs.

PROBLEM 4–35. Fig. 4–3 with a load of 1000 lbs. suspended at s.

PROBLEM 4–36. Fig. 4–4 if the tension in the cable, pt, is 1000 lbs.
Ans. 1730, 2000 lbs.

PROBLEM 4–37. Fig. 4–4 if the cable pt is vertical and the load is 1000 lbs.

In Problems 4–38 to 4–42 inclusive solve for the reactions at m and n by the method of moments. All structures are pin-connected. Weight of members neglected.

PROBLEM 4–38. Fig. 4–2 with a load of 1000 lbs. suspended at t.
Ans. 1250, 750 lbs.

PROBLEM 4–39. Fig. 4–2 with a load of 1000 lbs. suspended at u.
Ans. 1667, 1050 lbs.

Problem 4–40. Fig. 4–2 with a load of 1000 lbs. suspended at *s*.

Problem 4–41. Fig. 4–4 if the cable, *pt*, is at an angle of 60° with the vertical and has a tension of 1000 lbs.

Problem 4–42. Fig. 4–4 if the cable, *pt*, is horizontal and has a tension of 1000 lbs.

Solve problems 4–43 to 4–53 by any method. All structures are pin-connected. Weight of members neglected.

Problem 4–43. In Fig. 89, page 40, the length of the boom *mq* is 4.5 ft. and the length *pq* is 1.5 ft. If the load suspended at *q* is 1000 lbs. what are the reactions at *m* and *n*?

Problem 4–44. In Fig. 89 the length of the boom, *mq*, is 6 ft. and *pq* is 3 ft. If the load suspended at *q* is 1000 lbs. what are the reactions at *m* and *n*?

Problem 4–45. In Fig. 89 the load is 1000 lbs. What then are the reactions at *m* and *n*?

Problem 4–46. In Fig. 4–6 there is a counter weight of 3000 lbs. at *p* and a load of 1000 lbs. at *s*. What are the reactions at *m* and *n*? *Ans.* 500, 4040 lbs.

Problem 4–47. Fig. 1–1. What are the reactions at *m* and *n*?

Problem 4–48. Fig. 1–1. What are the tensions in *ts* and *su*?

Problem 4–49. Fig. 1–1. What are the forces at the joint *p*?

Problem 4–50. Fig. 1–1. What are the forces at the joint *u*?

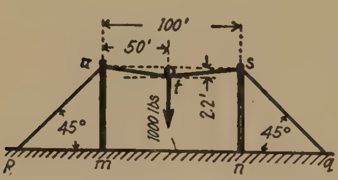


Fig. 4–7.

Problem 4–51. Fig. 4–7 shows a travelling hoist mounted on a cable, *us*. If the deflection of *us*, when the hoist is in the center of the span, is 2.2 ft. what is the tension in the cable, *us*? *Ans.* 11,350 lbs.

Problem 4–52. In the preceding problem what is the tension in the cables, *pu* and *sq*? *Ans.* 16,150 lbs.

Problem 4–53. If the hoist of Fig. 4–7 is loaded as shown but is 25 ft. from the left support, at which point the deflection of the cable *us* is 2.3 ft., what are the tensions in the guy cables, *pu* and *sq*?

PART V

In Problems 5–1 to 5–15 inclusive, disregard the weight of the members of the pin-connected structures.

Problem 5–1. Draw the space and force diagrams for the forces on the boom of Fig. 4–6.

Problem 5–2. In Fig. 4–6 there is a counter weight of 3000 lbs. at *p* and a load of 1000 lbs. at *s*. What are the forces on the boom, *ps*?

PROBLEM 5–3. In Fig. 4–6, Problem 5-2, what are the forces on the mast, *mn*?

PROBLEM 5–4. Draw space diagrams for each of the members of Fig. 4–6, showing the forces on them in magnitude and direction.

PROBLEM 5–5. In Fig. 86, page 40, if the load at the end of the boom is 2500 lbs. instead of the 1000 lbs. shown, what are the reactions?

PROBLEM 5–6. What are the forces on the mast of the preceding problem?

PROBLEM 5–7. In Fig. 87, page 40, if the load on the boom is 3300 lbs. instead of the 2000 lbs. shown, what are the reactions?

PROBLEM 5–8. What are the forces on the mast of the preceding problem?

PROBLEM 5–9. Draw the space diagrams for each of the members of Fig. 87 loaded as given in Problem 5–7.

PROBLEM 5–10. In Fig. 88, page 40, if the load suspended from the boom is 2500 lbs. instead of the 1000 lbs. shown, what are the reactions?

PROBLEM 5–11. What are the forces on the mast of the preceding problem?

PROBLEM 5–12. Draw the space diagram for each of the members of Fig. 88 when loaded as given in Problem 5–10.

PROBLEM 5–13. If in Fig. 87, page 40, the 2000-lb. load becomes a horizontal force to the right applied at the end of the boom, what are the reactions?

PROBLEM 5–14. What are the forces on the members of Fig. 87, page 40, when loaded as in the preceding problem?

PROBLEM 5–15. Draw the space diagram for each of the members of Fig. 87 when loaded as in Problems 5–13 and 5–14.

PROBLEM 5–16. If in Fig. 4–2 the weight of the boom is 600 lbs. and a load of 1000 lbs. is suspended at *t* what are the reactions at *m* and *n*? Disregard the weight of the cable, *mt*.

PROBLEM 5–17. If in the preceding problem the 1000-lb. load is suspended at *u* instead of at *t*, what are the reactions at *m* and *n*?

PROBLEM 5–18. If in the preceding problem the 1000-lb. load is suspended at *s* instead of at *t*, what are the reactions at *m* and *n*?

PROBLEM 5–19. If in Fig. 4–3 the weight of the boom is 600 lbs., the weight of the strut is 400 lbs. and a 1000-lb. load is suspended at *t*, what are the reactions?

PROBLEM 5–20. If in the preceding problem the 1000-lb. load is suspended at *u* instead of at *t*, what are the reactions?

PROBLEM 5–21. If in Problem 5–19 the 1000-lb. load is suspended at *s* instead of at *t*, what are the reactions?

PROBLEM 5–22. If in Fig. 4–4, the weight of the boom is taken as 100 lbs. per foot and the weight of the cable is neglected what are the reactions?

PROBLEM 5-23. If in Fig. 4-6 the weight of the mast is taken as 3200 lbs., of the boom as 2400 lbs. and of the strut as 800 lbs., in addition to the load and counter weight as shown, what are the reactions at m and n?

PROBLEM 5-24. What are the forces at q and u of the boom, ps, of the preceding problem?

PROBLEM 5-25. What are the forces on the mast, mn, of Problem 5-23?

PROBLEM 5-26. Draw the space diagrams for each member of the structure of Problem 5-23.

PROBLEM 5-27. If in Fig. 75, page 36, F is taken as 50 lbs. and F' as 150 lbs. what is the resultant by the "polygon of forces"?

PROBLEM 5-28. In Fig. 75, F is taken as 50 lbs. and F' as 150 lbs. respectively. Draw a space diagram showing these forces acting on a plank 6 ft. long. The 220-lb. force, acting at the left end, and the 125-lb. force, F, F' and the 100-lb. force acting respectively 1 ft., 2 ft., 3 ft., and 4 ft. from the left end. Draw the force and funicular polygons and locate the resultant with respect to the left end of the plank.

PROBLEM 5-29. If in addition to the forces on the plank of the preceding problem there is a force of 200 lbs. acting at the right end and inclined upward at an angle of 45° from the horizontal, where is the resultant located?

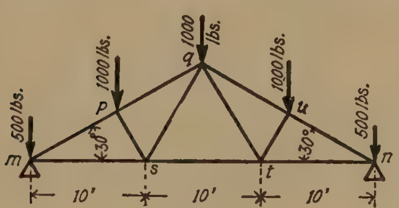


FIG. 5-1.

Solve Problems 5-30 to 5-37 inclusive, by graphical analysis.

PROBLEM 5-30. The roof truss of Fig. 5-1 is loaded as shown. What are the stresses in the members. *Ans.* Symmetrical 3000, 2500, 867, 867, 1730 lbs.

PROBLEM 5-31. The loads on the roof truss of Fig. 5-1 are each exactly doubled. What now are the stresses in the members?

PROBLEM 5-32. The loads at m, p, u and n are removed and the load at q is increased to 10,000 lbs. in Fig. 5-1. What now are the stresses in the members?

PROBLEM 5-33. The roof truss of Fig. 5-2 is loaded as shown. What are the stresses in the members?

PROBLEM 5-34. The loads at m, p, u, and n of Fig. 5-2 are removed and the load at q is increased to 10,000 lbs. What now are the stresses in the members?

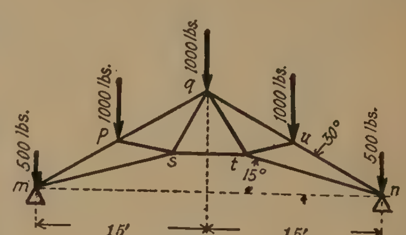


FIG. 5-2.

PROBLEM 5-35. The loads at m, and p of Fig. 5-2 are removed. What now are the stresses in the members?

PROBLEM 5-36. The bridge truss of Fig. 5-3 has vertical loads of 5000 lbs. at the points p and q. What are the stresses in the members?

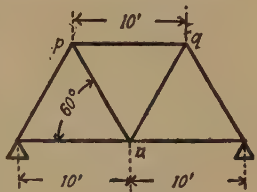


FIG. 5-3.

PROBLEM 5-37. The bridge truss of Fig. 5-3 has vertical loads of 5000 lbs. at p, q, and u. What are the stresses in the members?

PROBLEM 5-38. Assume that the truss shown in Fig. 5-1 has its left support attached in brickwork as is shown in Fig. 120, page 53. In addition to the vertical loads shown there is a wind load upon the left-hand slope of the truss which may be taken as concentrated at p perpendicular to the slope and equal to 2000 lbs. What now are the stresses in the members?

PROBLEM 5-39. Assume that the support, m, of the truss of Fig. 5-2 is anchored as in Fig. 120, page 53. In addition to the loads shown there is a wind load on the right-hand slope of the truss which may be taken as a concentrated load at u of 3000 lbs. perpendicular to the slope. What are the stresses in the members?

PROBLEM 5-40. Solve the truss of Fig. 5-1 for the stresses in the members by the method of sections.

PROBLEM 5-41. Solve for the stresses in the members of the truss of Fig. 5-2 by the method of sections.

PROBLEM 5-42. In the truss shown in Fig. 119, page 52, let there be vertical loads of 5000 lbs. at points m, n, p and q. What are the stresses in the members?

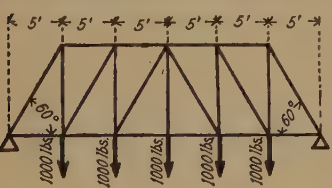


FIG. 5-4.

PROBLEM 5-43. The bridge truss shown in Fig. 5-4 is loaded as is shown. What are the stresses in the members?

PROBLEM 5-44. A shear leg crane as shown in Fig. 125 and 126, page 56, has the following dimensions: $m'm'' = 10$ ft.; $mn = 20$ ft.; angle $nmx = 60°$; and $onm = 30°$. What are the reactions?

PROBLEM 5-45. The shear leg crane of the preceding problem has the angle $mon = 45°$ instead of the angle onm as 30°. What now are the reactions?

PART VI

PROBLEM 6-1. If the weight of a block resting on a horizontal plane is 70 lbs. and the coefficient of friction between the block and the plane is 0.2, what is the magnitude of a horizontal force, F, if motion is impending? *Ans.* 14 lbs.

PROBLEM 6-2. If the force, F, of the preceding problem is inclined upward to the right at an angle of 30° with the horizontal, what then is its magnitude if motion is impending? *Ans.* 14.5 lbs.

PROBLEM 6-3. If the force, F, of Problem 6-1 is inclined downward to the right at an angle of 30° with the horizontal, what is its magnitude if motion is impending? *Ans.* 18.25 lbs.

PROBLEM 6–4. If the weight of a block resting on a plane which makes an angle of 30° with the horizontal is 70 lbs. and the coefficient between the block and the plane is 0.2, what is the magnitude of a force, F, parallel to plane if motion up the plane is impending? *Ans.* 47.1 lbs.

PROBLEM 6–5. If the force, F, of the preceding problem is inclined upward at an angle of 60° with the horizontal, what then is its magnitude if motion up the plane is impending (Fig. 6–1)? *Ans.* 48.8 lbs.

PROBLEM 6–6. If the force, F, of Problem 6–4 is horizontal, what is its magnitude if motion up the plane is impending? *Ans.* 61.5 lbs.

PROBLEM 6–7. A block weighing 100 lbs. is resting on a horizontal plane. A horizontal force, F, of 10 lbs. is applied to the block. If the coefficient of friction between the block and the plane is 0.2 what is the friction force?

PROBLEM 6–8. A block weighing 100 lbs. is resting on a plane which is inclined 30° with the horizontal. If the coefficient of friction between the block and the plane is 0.2 what are the magnitude and direction of a horizontal force, F, if motion down the plane is impending?

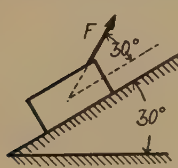


FIG. 6–1.

PROBLEM 6–9. If the coefficient of friction between the block and the plane of Fig. 6–1 is 0.2 and the weight of the block is 100 lbs. what is F if motion down the plane is impending? *Ans.* 42.7 lbs.

PROBLEM 6–10. If in the preceding problem the force, F, is horizontal, what are its magnitude and direction if motion down the plane is impending?

PROBLEM 6–11. A block weighing 100 lbs. is resting on a plank. Assuming that the plank is rigid and that the coefficient of friction between the block and plank is 0.2 to what angle may the plank be inclined to the horizontal before motion is impending? *Ans.* 11° 19′.

PROBLEM 6–12. If the block shown in Fig. 6–1 weighs 100 lbs. and the angle of friction is 15° what is the magnitude of F if motion up the plane is impending?

PROBLEM 6–13. For the conditions of the preceding problem, what are the magnitude and direction of F if motion down the plane is impending?

PROBLEM 6–14. Draw the space diagrams for Fig. 6–2 if P weighs 2000 lbs. and the angle of friction for all surfaces is 15° and an upward motion of P is impending.

PROBLEM 6–15. Fig. 6–2. Block P weighs 2000 lbs. The angle of friction for all surfaces is 15°. If the weight of wedge Q is neglected what is the magnitude of the force F if upward motion of P is impending? *Ans.* 3460 lbs.

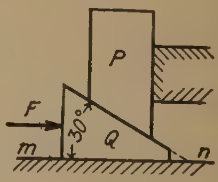


FIG. 6–2.

PROBLEM 6–16. Draw the space diagrams for Fig. 6–2 if the conditions are as in Problem 6–14 but motion of P downward is impending.

PROBLEM 6–17. If in Fig. 6–2 the block P weighs 2000 lbs.; the angle of friction of all surfaces is 15° and the weight of Q is negligible, what is F if motion of P downward is impending?

PROBLEM 6–18. Draw the space diagrams for the block and wedge of Fig. 6–3 if the weight of P is taken as 1000 lbs., the wedge Q as weightless, angle of friction of all surfaces as 15°, and an upward motion of P is impending.

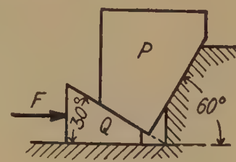


FIG. 6–3.

PROBLEM 6–19. What is the magnitude of the force F of Fig. 6–3 if the conditions are as in the preceding problem?

PROBLEM 6–20. Draw the space and force diagrams for the block and wedge of Fig. 6–3 if the conditions are as in Problem 6–18 excepting that the impending motion of P is downward.

PROBLEM 6–21. What are the magnitude and direction of F of Fig. 6–3 under the conditions of the preceding problem?

PROBLEM 6–22. Draw the space diagrams for the block and wedge of Fig. 6–4 if the weight of P is taken as 1000 lbs., the weight of Q as negligible, the angle of friction of all surfaces as 15°, and motion of P to the right is impending.

PROBLEM 6–23. What is the magnitude of F of Fig. 6–4 under the conditions of the preceding problem? *Ans.* 460 lbs.

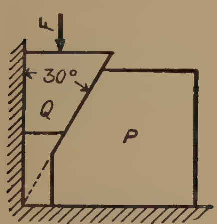


FIG. 6–4.

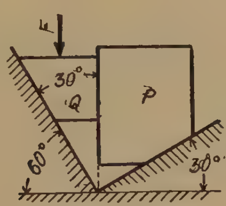


FIG. 6–5.

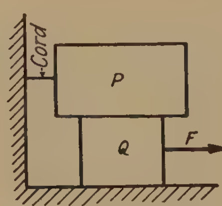


FIG. 6–6.

PROBLEM 6–24. What is the magnitude of F of Fig. 6–5 to cause motion of P up the plane to be impending if the weight of P is 1000 lbs., the weight of Q is negligible and the angle of friction of all surfaces is 15°? *Ans.* 1730 lbs.

PROBLEM 6–25. If F of Fig. 6–5 is at an angle of 30° with the vertical (parallel with the wall) other conditions as in the preceding problem, what is its magnitude to cause motion of P up the plane to be impending? *Ans.* 1268 lbs.

PROBLEM 6–26. If in Fig. 6–6 P is tied to the wall and weighs 1000 lbs., the coefficient of friction of all surfaces is 0.2, and the weight of Q is negligible, what is the magnitude of F to cause motion of Q to be impending?

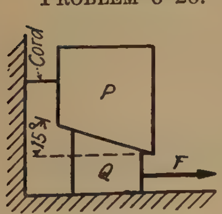


FIG. 6–7.

PROBLEM 6–27. If in Fig. 6–6 the weight of Q is 500 lbs., and the other conditions are as in the preceding problem what is F?

PROBLEM 6–28. If in Fig. 6–7 the weight of P is taken as 1000 lbs., the weight of Q as negligible, and the angle of friction of all surfaces as 30°, what is F if motion of Q to the right is impending?

Problem 6–29. If in Fig. 6–7 the weight of Q is taken as 500 lbs. and other conditions as in the preceding problem what is F if motion of Q to the right is impending? *Ans.* 1865 lbs.

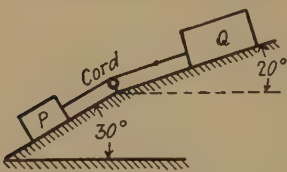


Fig. 6–8.

Problem 6–30. In Fig. 6–8 blocks P and Q are resting on inclined planes as shown. They are connected by a cord running over a frictionless pulley so that the pull, if any, of the cord on the blocks is parallel to their respective planes. If block P weighs 30 lbs. and the coefficient of static friction under this block is 0.20; and the weight of Q is 289.4 lbs. and the coefficient of static friction under it is 0.4, what is the tension in the cord? What is the friction force under P? The friction force under Q? *Ans.* 9.8, 5.2, 108.6 lbs.

PART VII

Problem 7–1. An automobile is brought up to a speed of 30 miles per hour in 8 sec. What is the acceleration? What is the distance travelled? *Ans.* 5.5 ft./sec.^{2}, 176 ft.

Problem 7–2. An automobile travelling at a rate of 30 miles per hour is brought to a stop in a distance of 100 ft. What is the average acceleration? *Ans.* 9.66 ft./sec.^{2}.

Problem 7–3. A racing automobile is uniformly retarded over a distance of 22 miles. The velocity is then 20 miles per hour. If 5 additional minutes are needed to bring it to a complete stop at the same rate of retardation what was the initial velocity. *Ans.* 104.5 mi./hr.

Problem 7–4. A train is brought to rest from a speed of 70 miles per hour at a constant retardation of 2000 miles per hour per hour. What is the distance travelled? *Ans.* 1.22 miles.

Problem 7–5. A train travelling at a rate of 30 miles per hour is uniformly retarded to a speed of 20 miles per hour in a distance of $\frac{1}{2}$ mile. In what additional distance can it be brought to rest at the same rate of retardation? *Ans.* 0.4 mile.

Problem 7–6. An interurban car starts from rest at one station and is uniformly accelerated to a speed of 50 miles per hour in 4 minutes. It travels at a constant velocity for a certain distance when the brakes are applied and it is brought to rest in 3 minutes, at a uniform rate of retardation. If the total elapsed time is 15.5 minutes how far did the car travel? *Ans.* 10 miles.

Problem 7–7. A wheel is 5 ft. in radius and is revolving at N revolutions per minute. What is (a) the linear velocity of a point on its circumference in feet per second? (b) the angular velocity in radians per second?

Problem 7–8. The wheel of the preceding problem is increased in speed to N' revolutions per minute in T seconds. What is (a) the linear acceleration of a point in its circumference in feet per second per second? (b) what is the angular velocity in radians per second per second?

Problem 7–9. A flywheel is brought from rest to a speed of 60 r.p.m. in $\frac{1}{2}$ min. What are the average angular acceleration and the number of revolutions required?

PROBLEM 7–10. A pulley rotating at 120 r.p.m. comes to rest in 3 min. What are the angular acceleration and the number of revolutions required?

PROBLEM 7–11. If the peripheral speed of a pulley is increased 300 ft. per min. in 4 sec. under an angular acceleration of 3.75 radians per sec. per sec. what is the diameter of the pulley? *Ans.* 8 ins.

PROBLEM 7–12. The r.p.m. of a flywheel is uniformly decreased from 200 to 120 r.p.m. in 50 turns. In how many additional turns can the wheel be brought to rest at the same rate of retardation? *Ans.* 28 turns.

PROBLEM 7–13. The r.p.m. of a flywheel is uniformly decreased by 100 in 180 turns during 2 min. What is the time required to bring the wheel to a complete stop at the same rate of retardation? *Ans.* 0.8 min.

PROBLEM 7–14. The r.p.m. of a flywheel is uniformly decreased by 180 in 60 turns during 30 sec. In how many additional turns can it be brought to rest at the same rate of retardation? *Ans.* 1.25 turns.

PROBLEM 7–15. A cylindrical drum 3 ft. in diameter is rotated on its axis by pulling a rope wound around it. If the linear acceleration of a point on the rope is 64.4 ft. per sec. per sec. what will be the r.p.m. of the drum if started from rest, at the end of 5 sec.? *Ans.* 2050 r.p.m.

PART VIII

PROBLEM 8–1. A body weighs 100 lbs. at standard locality. What does it weigh at a locality where the acceleration of gravity is 31 ft. per sec. per sec.? Does the mass change? Could the change in weight be shown by a spring balance? By a beam scales?

PROBLEM 8–2. A force of 1 lb. produced by the turning effort of an engine is measured at Baltimore where the value of g is 32.16. The same force is measured at a locality where g is 32.1. Will the scales indicate the same, more or less force if (*a*) a beam scales is used? (*b*) a spring scales is used?

PROBLEM 8–3. A quantity of sugar weighing 10 lbs. at Baltimore is weighed again at the other locality of the preceding problem. Will the same, more or less sugar be weighed out (*a*) if a beam scales is used? (*b*) if a spring balance is used?

PROBLEM 8–4. Which has the larger weight, a cubic foot of iron of specific gravity 8 or 8 cu. ft. of water? Which has the larger mass?

PROBLEM 8–5. What principle is applied when one tightens the head of a hammer by pounding on the handle?

PROBLEM 8–6. If the water pressure in the city water mains is 40 lbs. per sq. in. and this is supplied by a water tower, how high is the tower above the main?

PART IX

PROBLEM 9–1. An elevator cage weighing 500 lbs. is accelerated downward. If the tension in the cable is 475 lbs. what is the acceleration?
Ans. 1.61 ft. per sec. per sec.

PROBLEM 9–2. The tension in the cable of the preceding problem is 525 lbs. What is the acceleration?

PROBLEM 9–3. A train of 3 cars each weighing 30 tons is under a draw-bar pull of 1500 lbs. If the total resistances may be taken as 10 lbs. per ton, what is the acceleration and the force on the coupling of the last two cars?

Ans. 0.107 ft. per sec. per sec., 500 lbs.

PROBLEM 9–4. An elevator cage weighing 500 lbs. is accelerated downward with an acceleration of 5 ft. per sec. per sec. What is the tension in the cable?

Ans. 422 lbs.

PROBLEM 9–5. Four blocks are resting on a horizontal plane and are tied together with cords. If the coefficient of kinetic friction of all surfaces is 0.1 and the first block is under a horizontal pull of 50 lbs. what is the tension in the cord connecting the second and third blocks if the blocks weigh 32.2, 64.4, 96.6 and 128.8 lbs. respectively.

Ans. 35 lbs.

PROBLEM 9–6. An automobile has a wheel base of 120 ins., and the center of gravity is 15 ins. from the ground and midway between the axles. If the car weighs 2500 lbs. what are the reactions on the rear wheel if the acceleration is 3.22 ft. per sec. per sec.?

Ans. 1281 lbs.

PROBLEM 9–7. An automobile with four-wheel brakes has an uniform retardation of 19.32 ft. per sec. per sec. If during this retardation the normal reactions on both front and rear wheels are equal, how far from the rear wheels is the center of gravity if it is in a plane 3 ft. above the ground? Wheel base is 10 ft.

Ans. 3.2 ft.

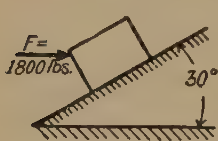


FIG. 9–1.

PROBLEM 9–8. Fig. 9–1 shows a block on a 30° plane and under the action of a horizontal force of 1800 lbs. If the coefficient of kinetic friction between the block and the plane is 0.3 and the block is accelerated up the plane at a rate of 8.05 ft. per sec. per sec. what is the weight of the block?

Ans. 1290 lbs.

PROBLEM 9–9. The automobile shown in Fig. 9–2 has a weight of 3220 lbs. and is ascending a 15° grade with an uniform retardation of 10 ft. per sec. per sec. What are the normal reactions at the wheels?

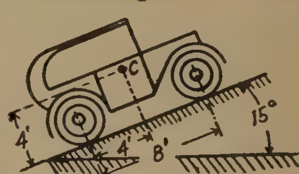


FIG. 9–2.

PROBLEM 9–10. A trolley car weighing 10 tons has two trucks spaced 15 ft. apart. The center of gravity is midway between the trucks and 5 ft. above the rails. The car has brakes on the front trucks only. If the coefficient of friction between wheels and rails is 0.5, what is the shortest distance in which the car can be stopped at uniform retardation from a speed of 30 miles per hour?

Ans. 100.2 ft.

PROBLEM 9–11. The car of the preceding problem is equipped with brakes on both trucks. Assuming the same conditions otherwise, in what distance can the car be stopped?

PROBLEM 9–12. Assuming the same conditions as in Problem 9–10 except that the brakes are on the rear trucks in what distance can the car be stopped?

PROBLEM 9–13. A casting 8 ft. by 3 ft. by 3 ft. is standing on end on a flat car with its sides parallel to the sides of the car. If the casting is homogeneous and

the coefficient of friction between the car and casting is 0.3, at what acceleration of the car will the casting slide, or tip, and which will it do?

PROBLEM 9-14. If the casting of the preceding problem is standing as is given in that problem what must the coefficient of friction between car and block be to just insure tipping when the car is accelerated? What is the necessary acceleration?

Ans. 0.374, 12.1 ft. per sec. per sec.

PROBLEM 9-15. In Fig. 147, page 76, if L is 2 ft., L' is 3 ft. and the kinetic coefficient of friction is 0.1 what is the acceleration necessary to cause the block to tip about the point m? The center of gravity is 3 ft. above mn.

Ans. 25.76 ft. per sec. per sec.

PROBLEM 9-16. In Fig. 9-1 the weight of the block is taken as 400 lbs., the kinetic coefficient of friction between the block and the plane as 0.2 and the horizontal force F as 1000 lbs., what acceleration is produced?

Ans. 40 ft. per sec. per sec.

PROBLEM 9-17. In Fig. 9-3 if the weight of P is 100 lbs. and the coefficient of kinetic friction under it is 0.1, the pulley is frictionless and the weight of Q is 120 lbs., what is the tension in the cord? Draw space diagrams for each block. *Ans.* 60 lbs.

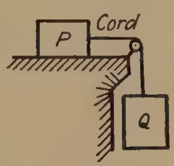


FIG. 9-3.

PROBLEM 9-18. A car weighing 40,000 lbs. comes to rest from a velocity of 10 miles per hour in 10 sec. If the center of gravity is 8 ft. above the rails, 15 ft. from the rear truck and 20 ft. from the front truck, what are the normal reactions on each truck if the retarding force is constant and is friction at the rails only?

Ans. 13,659, 26,343 lbs.

PROBLEM 9-19. In Fig. 9-4 block P weighs 100 lbs., block Q, 200 lbs., and block S, 300 lbs. The kinetic coefficient of friction between all surfaces is 0.2. The blocks Q and S are to be accelerated 8.05 ft. per sec. per sec. by a horizontal force, F, so located along the link, mn, that the link remains in a vertical position. Draw the space and force diagram for each block.

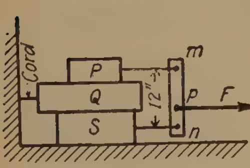


FIG. 9-4.

PROBLEM 9-20. In Problem 9-19 what is F and where is it located?

Ans. 300 lbs.; 1.8 ins. above n.

PROBLEM 9-21. The center of gravity of an automobile is 3.5 ft. from the ground and midway between the wheels. The wheel base is 120 ins. Coefficient of friction at the rear wheels and road-bed is $\frac{1}{3}$. What is the maximum tractive force which could be exerted when the car is accelerated from rest if the weight of the car is 2000 lbs.? *Ans.* 377 lbs.

PROBLEM 9-22. The center of gravity of an automobile is 5 ft. above the ground and midway between the wheels. If the wheel base is 6 ft. what forward acceleration would cause the car to tip around the rear wheels?

Ans. 19.3 ft. per sec. per sec.

PROBLEM 9-23. What coefficient of friction would be necessary in Problem 9-22 to produce the necessary acceleration? *Ans.* 0.6.

PART X

Problem 10–1. The stroke of a Scotch Yoke engine is 4 ins. and the weight of reciprocating parts is 10 lbs. If the maximum inertia force is + 560 lbs. what is the r.p.m.? *Ans.* 990 r.p.m.

Problem 10–2. The cross-head of a Scotch Yoke engine is displaced 3.42 ins. to the left of mid-position. If the stroke of the engine is 20 ins. and the r.p.m. is 200 what is the acceleration in that position? *Ans.* 125 ft. per sec. per sec.

Problem 10–3. If the weight of the reciprocating parts of the engine of the preceding problem is 20 lbs., what is the inertia force in the position given; and at dead center positions? *Ans.* 77.7; ± 226 lbs.

Problem 10–4. If the r.p.m. of the engine of the preceding problem is increased to 1200 r.p.m., what then is the inertia force in the position given; and at the dead center positions? *Ans.* 2780; ± 8014 lbs.

Problem 10–5. What size spring (pounds per inch of compression or extension) could be used to balance the inertia forces of the engine of Problem 10–3 if the r.p.m. is 400? *Ans.* 90.4 lbs. per inch.

Problem 10–6. A conventional steam engine has a crank radius of 12 ins. and a connecting-rod length of 48 ins. The weight of reciprocating parts is 60 lbs. and the r.p.m. is 100. What are the inertia forces at the dead center positions? *Ans.* 255, −153 lbs.

Problem 10–7. A conventional 2-cylinder engine has cranks located as shown in Fig. 156, page 89. The crank is 2 ins., connecting rod 8 ins., r.p.m. 1500, weight of reciprocating parts 10 lbs. What are the reactions in the bearings if the dimensions are as given in that figure? When the cranks are 180° from the position shown?

Problem 10–8. What are the reactions on the bearings if a four-cylinder engine with the dimensions of the preceding problem has its cylinders arranged as in Fig. 157, page 89?

PART XI

Problem 11–1. An eye rod with the dimensions shown in Fig. 164, page 96, weighs 20 lbs. If it is revolving at a speed of 1000 r.p.m. what is the centripetal force and where does it act? *Ans.* 17,000 lbs.

Problem 11–2. If the turbine rotor of Fig. 166, page 96, is revolving at 1200 r.p.m., has a weight of 1200 lbs., and its center of gravity is $\frac{1}{8}$ in. away from the axis of rotation, what are the total reactions on the bearings, P and Q, in the position shown? *Ans.* 2465 lbs.

Problem 11–3. If the turbine rotor of the preceding problem is revolving at such a speed that the total reactions on each bearing is changing from 1200 lbs. to zero during each half-revolution, what is its r.p.m.? *Ans.* 531 r.p.m.

PROBLEM 11-4. A turbine rotor with a weight of 15,000 lbs. has a center of gravity located with respect to two bearings P and Q as is shown in Fig. 11-1. If the center of gravity is 0.01 in. away from the axis of rotation what is the maximum pressure on either of the bearings at 1800 r.p.m.?

Ans. 17,382 lbs. on Q.

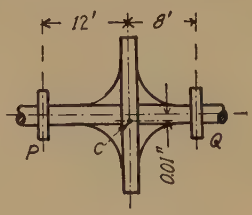


FIG. 11-1.

PROBLEM 11-5. If the height, L (see fly ball governor, Fig. 167, page 97) is 16 ins. what is the r.p.m.?

Ans. 47 r.p.m.

PROBLEM 11-6. A motor cycle and rider weighing 800 lbs. are rounding a curve which is banked at 30°. If the radius of the track is 64.4 ft. and the cyclist's velocity is 32.2 ft. per sec., what is the value of the friction force and in which direction does it act?

Ans. 53.6 lbs. up plane.

PROBLEM 11-7. It is desired to bank an automobile race course curve so that, at a speed of 100 miles per hour, there will be no lateral pressure on the tires of a car rounding it. If the weight of a racing car may be taken at 3000 lbs., location of the center of gravity 2 ft. above the track and midway between wheels, coefficient of friction between tires and road bed 0.667, tread of car 40 ins., wheel base 120 ins., and the radius of curvature 250 ft., what is the angle of the bank?

PROBLEM 11-8. The radius of a vertical loop track is 22 ft. If the center of gravity of the car used is 2 ft. from the track what is the minimum velocity necessary at the top of the loop that the car will remain on the track?

Ans. 25.4 ft. per sec.

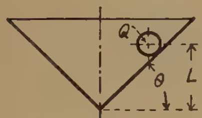


FIG. 11-2.

PROBLEM 11-9. If in Fig. 11-2 the angle θ is 45° what velocity in feet per second is necessary to keep the ball, Q, travelling around the track at a vertical elevation, L, of 12.5 ft.?

Ans. 20 ft. per sec.

PROBLEM 11-10. If in Fig. 11-2 the angle θ is 30° what velocity in feet per second is necessary to keep the ball Q travelling around the track at an elevation, L, of 12.5 ft.?

PROBLEM 11-11. The two bodies, P and Q, of Fig. 11-3 are arranged symmetrically with respect to om as shown. If they each weigh 100 lbs. and are rotated in their plane about o at 1800 r.p.m. what is the tension in om?

Ans. 661,000 lbs.

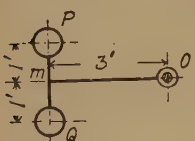


FIG. 11-3.

PROBLEM 11-12. In the preceding problem what is the tension in the arm connecting P and Q?

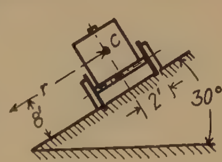


FIG. 11-4.

PROBLEM 11-13. Assuming a box car with the dimensions shown in Fig. 11-4 rounding a curve banked 30° with a radius of curvature, r, measured as shown, of 200 ft., at what speed will this car tip from the rails?

Ans. 67.8 ft. per sec.

PROBLEM 11-14. A shaft revolving on its own axis is mounted on two bearings 10 ft. apart. A rotor weighing 200 lbs. attached to the shaft 3 ft. from the left-

hand bearing, has its centroid $\frac{1}{8}$ in. away from the shaft axis. Another rotor, located 4 ft. to the right of the first, and weighing 300 lbs., has its centroid $\frac{1}{4}$ in. away from the shaft axis, and in the plane fixed by the centroid of the first rotor and the shaft axis, but on the opposite side of the axis. What are the kinetic reactions at 3000 r.p.m.? *Ans.* 1280, 11,520.

Problem 11–15. How can the kinetic reactions of the rotors of the preceding problem be reduced to zero by adding another weight to the shaft?

Problem 11–16. A shaft carries two pulleys. Pulley Q, at the left, weighs 200 lbs., and its centroid is located 0.5 in. from the shaft axis. Pulley P, 2 ft. to the right, weighs 300 lbs., and its centroid is 0.1 in. away from the shaft axis in the plane fixed by the centroid of pulley Q and the shaft axis, but on the opposite side of the axis. The shaft is turning at 500 r.p.m. Show that the centrifugal forces may be considered as a force and a couple and calculate their magnitudes.

Problem 11–17. It is desired to place a single weight on the shaft of Problem 11–16 to balance the centrifugal forces on P and Q. This weight is to be located 6 ins. away from the axis. What must be the weight and where must it be placed with respect to pulley Q?

Problem 11–18. Will the same weight balance the centrifugal forces of P and Q if the shaft is turned 180° from the first position?

Problem 11–19. Where must the weight of Problem 11–17 be placed if the pulleys P and Q are reversed in position on the shaft but retain the weights and displacements of centroids of Problem 11–16?

PART XII

Problems 12–1 to 12–10 inclusive refer to Fig. 12–1 which represents three small bodies weighing 0.161 lb. each and mounted on a rigid weightless wire at the distances shown from the point, o, around which they are revolving.

Fig. 12–1.

Problem 12–1. If the system is revolving about o at 240 r.p.m. what is the angular velocity of m? of n? of p?

Ans. 25.1 radians per sec.

Problem 12–2. What is the linear velocity of each body about o if they are revolving at 240 r.p.m.?

Ans. 25.1, 50.2, 75.3 ft. per sec.

Problem 12–3. If the r.p.m. of the bodies m, n, and p about o is raised from 240 to 480 in 2 revolutions what is the angular acceleration?

Ans. 75.3 radians per sec. per sec.

Problem 12–4. What is the linear acceleration of each body of the preceding problem about o? *Ans.* 75.3, 150.7, 226.1 ft. per sec. per sec.

Problem 12–5. What is the inertia force of each body at this acceleration? *Ans.* 0.376, 0.753, 1.131 lbs.

Problem 12–6. What is the moment about o of the inertia force of m? of n? of p? *Ans.* 0.376, 1.51, 3.39 lb.-ft.

PROBLEM 12–7. What moment (produced by a force applied at any distance, L, from o), would balance the inertia forces of Problem 12–5? *Ans.* 5.28 lb.-ft.

PROBLEM 12–8. Assume that the three bodies, m, n and p, to be concentrated at a distance, r_g, from o instead of being at the distances of 1, 2 and 3 ft. respectively. What is the force required to give their combined masses at radius, r_g, the linear acceleration corresponding to the same angular acceleration found in Problem 12–3, namely 75.3 radians per sec. per sec.? *Ans.* 1.13 r_g.

PROBLEM 12–9. What is the moment of the force of the preceding problem about o?

PROBLEM 12–10. What is the radius, r_g, of Problem 12–8? *Ans.* 2.16 ft.

PROBLEM 12–11. Suppose that the bodies of Fig. 12–1 be taken as weighing 0.322 lb., at what distance, r_g, from o, could they be concentrated and have the same angular acceleration as that given in Problem 12–8 under the same turning moment? *Ans.* 2.16 ft.

PROBLEM 12–12. A flywheel with a moment of inertia of 120 units of mass feet squared is accelerated under a constant turning moment of 240 lb.-ft. What is the angular acceleration? *Ans.* 2 radians per sec. per sec.

PROBLEM 12–13. What is the r.p.m. of the flywheel of the preceding problem after 15 sec. if starting from rest? *Ans.* 286 r.p.m.

PROBLEM 12–14. A rotor revolving at 1000 r.p.m. freely, comes to rest in 15 sec. under a constant resisting moment of 125 lb.-ft. What is its moment of inertia? *Ans.* 17.8 units of mass-ft.^{2}

PROBLEM 12–15. A flywheel with a moment of inertia of 600 units of mass-ft. squared is caused to accelerate due to a turning moment of 150 lb.-ft. If there is a total resisting moment of 50 lb.-ft. what is the r.p.m. at the end of 1 minute if starting from rest? *Ans.* 95.5 r.p.m.

PROBLEM 12–16. Fig. 12–2 shows a flywheel which is caused to revolve by virtue of a load of 100 lbs. hanging from a rope wound around a drum 4 ft. in diameter. If the moment of inertia of the rotating masses is 5 units of mass-ft. squared, what is the r.p.m. of wheel after the weight has dropped 10 ft., starting from rest? *Ans.* 100 r.p.m.

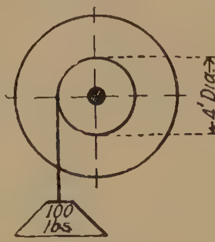


FIG. 12–2.

PROBLEM 12–17. A rotor weighing 1200 lbs. and having a radius of gyration of 16 ins. is revolving at 1000 r.p.m. It is brought to rest in 5 minutes by a constant resisting moment. What if the resisting moment? *Ans.* 23.1 ft.-lbs.

PROBLEM 12–18. A rectangular prism 6 ins. × 8 ins. × 10 ins. is formed of cast iron which weighs 0.26 lb. per cu. in. What is the mass moment of inertia of this prism about a polar centroidal axis perpendicular to the 6 × 8 face? *Ans.* 32.4 units of mass-in.^{2}

PROBLEM 12–19. The moment of inertia of a sphere about a diameter is $\frac{2}{5}Mr^2$. What is the mass moment of inertia of a sphere 24 ins. in diameter about a diameter if it weighs 322 lbs,? *Ans.* 4 units of mass-ft.^{2}

PROBLEM 12–20. What is the area moment of an equilateral triangle of side L about a centroidal polar axis? *Ans.* $\dfrac{L^4\sqrt{3}}{48}$.

PROBLEM 12–21. What is the radius of gyration of an equilateral triangle of side L about a centroidal polar axis? *Ans.* $\dfrac{L}{2\sqrt{3}}$.

PROBLEM 12–22. What is the centroidal polar area moment of inertia of the shaded figure of Fig. 12–3? *Ans.* 11.98 ft.^{4}

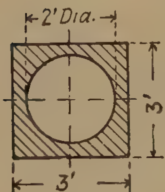


FIG. 12–3.

PROBLEM 12–23. If the shaded figure of Fig. 12–3 represents the cross-section of a solid 10 ft. long weighing 3220 lbs., what is the mass moment of inertia of the solid about a polar centroidal axis perpendicular to that cross-section? *Ans.* 204 units of mass-ft.^{2}

PROBLEM 12–24. What is the mass moment of inertia of the sphere 24 ins. in diameter and weighing 322 lbs. about a tangent. *Ans.* 14 units of mass-ft.^{2}

PROBLEM 12–25. The area moment of inertia of the rectangle shown in Fig. 12–4 about $x'x'$ is 576 ft.^{4} and its area is 48 sq. ft. What is its moment about $x''x''$? *Ans.* 1344 ft.^{4}

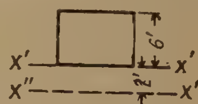


FIG. 12–4.

PROBLEM 12–26. What is the area moment of inertia of the shaded figure of Fig. 3–6 about an axis through o parallel to the 7-in. side? *Ans.* 633.67 in.^{4}

PROBLEM 12–27. What is radius of gyration of the figure of Fig. 3–6 about an axis through o parallel to the 7-in. side? *Ans.* 4.45 ins.

PROBLEM 12–28. A cast-iron flywheel (weight 450 lbs. per cu. ft.) 4 ft. in diameter has a rim 2 ins. thick and 12 ins. wide. To its axle is fastened a 20-in. pulley around which a cord passes. If a 100-lb. force is applied to the end of this cord what is the angular acceleration of the flywheel? Neglect the mass of the axle, hub, spokes, pulley and shaft and friction and windage. *Ans.* 0.786 radian per sec.^{2}

PROBLEM 12–29. If a load of 100 lbs. is substituted for the 100-lb. force in the preceding problem what is the acceleration produced? *Ans.* 0.765 radian per sec.^{2}

PROBLEM 12–30. An elevator with a 200-ft. lift weighs 1000 lbs. It is counterbalanced by a cable 240 ft. long running over a 6-ft. diameter pulley at the top of the shaft to a 1000-lb. counter-weight. Cable weighs 2 lbs. per foot. Center of pulley is 15 ft. above top of elevator at top of travel. What must be the necessary tangential force exerted at a 7-in. crank to produce an acceleration of 10 ft. per sec. per sec. downward from the top of travel if the weight of the pulley, shaft and crank be taken as 160 lbs. and the radius of gyration about the shaft axis to be 2.5 ft.? *Ans.* 6197 lbs.

PROBLEM 12–31. A solid cylinder of aluminum has the same weight as a hollow cylinder of cast iron. If the outside diameters are the same, which has the greater mass moment of inertia with respect to a polar centroidal axis or are they equal? Why?

PART XIII

PROBLEM 13–1. Let the block shown in Fig. 6–1 weigh 100 lbs. and be moved up the 30° plane at uniform velocity under the action of the force, F. If there is no friction what are the useful work and the supplied work?

PROBLEM 13–2. If friction be considered in the preceding problem and the kinetic coefficient is 0.2, what is the useful work, the lost work, and the supplied work?

PROBLEM 13–3. What are the mechanical efficiencies of the processes in each of the two preceding problems?

PROBLEM 13–4. A piston of an engine with a 4-in. stroke is acted upon by an average effective pressure of 900 lbs. throughout the stroke. What is the work done? *Ans.* 300 ft.-lbs.

PROBLEM 13–5. A shaft revolving at 120 r.p.m. is under a torque of 1500 lb.-ft. How much work will it transmit per minute?

PROBLEM 13–6. If 80 per cent of the work transmitted by the shaft of the preceding problem is utilized in raising a body weighing 100 lbs., how many feet will that body be raised in 2 sec.?

PROBLEM 13–7. A single-cylinder single-acting steam engine has an average effective force of 550 lbs. on its piston during a working stroke. A single-cylinder single-acting engine has one working stroke per revolution. If 10 per cent of the work done in the cylinder is lost what is the torque on the shaft?

PROBLEM 13–8. A shaft P drives another shaft Q by means of a belt. The belt pulley on P is 12 ins. in diameter and on Q is 48 ins. If the torque delivered to the driving pulley is 200 lb.-ft. with no losses what is the torque on Q?

PROBLEM 13–9. If the belt tension leading off of the driving pulley of the preceding problem is 50 lbs. what is the belt tension leading onto the pulley? *Ans.* 450 lbs.

PROBLEM 13–10. A spring is 16 ins. long when free. A force of 75 lbs. is required to compress it 1 in., the force required being directly proportioned to the amount of linear compression. How much work is done when compressing it from 16 to 14 ins.? When it is compressed from 14 to 12 ins.?

PROBLEM 13–11. An automobile weighing 1800 lbs., travelling at a speed of 30 miles per hour, is stopped in 100 ft. What is its kinetic energy? *Ans.* 54,200 ft.-lbs.

PROBLEM 13–12. If 25 per cent of the kinetic energy of the auto of the preceding problem is absorbed by windage; the balance by the brake drums on the rear wheels; the rear wheels are 32 ins. in diameter and do not slip on the road bed; what torque is exerted on the brake drums? *Ans.* 532 lb.-ft.

PROBLEM 13–13. An automobile running at 30 miles per hour strikes a 15 per cent grade. Neglecting friction, windage, etc., what distance will the car travel up the hill because of its kinetic energy and a constant traction force of 150 lbs. at the rear wheels? *Ans.* 452 ft.

Problem 13–14. Under the conditions of the preceding problem what is the speed of the car when 200 ft. up the grade?

Problem 13–15. What was the kinetic energy of an auto weighing 2000 lbs. if brought to rest in 50 ft. by a friction force between tires and road bed of 400 lbs. Neglect windage. *Ans.* 20,000 ft.-lbs.

Problem 13–16. What was the velocity before braking of the car in the preceding problem? *Ans.* 25.4 ft. per sec.

Problem 13–17. Using the linear relation what is the kinetic energy of a wheel 6 ft. in diameter, radius of gyration 2.75 ft., weight 2000 lbs., when turning at 1200 r.p.m.? *Ans.* 37,000 ft.-lbs.

Problem 13–18. If a tangential force of 30 lbs. is applied to the rim of the wheel of the preceding problem how many revolutions will the wheel make before coming to rest? *Ans.* 65.3 turns.

Problem 13–19. A flywheel weighing 2000 lbs. and with a radius of gyration of 2.16 ft. is increased in speed from 490 to 510 r.p.m. What is the change in its kinetic energy?

Problem 13–20. If the increase in speed of the flywheel of the preceding problem was caused in $\frac{1}{4}$ turn by a tangential force applied to a 6-in. crank, what was the torque supplied?

Problem 13–21. If the flywheel of the second problem preceding was brought from rest to a speed of 500 r.p.m. in 100 turns what was the necessary average tangential force if applied to a 6-in. crank?

Problem 13–22. A flywheel rotating at 600 r.p.m. has a mass moment of inertia of 800 units of mass-ft. squared. If the speed drops 10 per cent how much work is released? *Ans.* 302,000 lb.-ft.

Problem 13–23. A flywheel with a radius of gyration of 3 ft. and weighing 3000 lbs. changes in speed from 300 to 270 r.p.m. in a $\frac{1}{4}$ turn. What torque is delivered during this speed change? *Ans.* 50,300 ft.-lbs.

Problem 13–24. There is a deficiency of torque at the crank of an engine of 1000 lb.-ft. when the crank passes through an angle of 20°; (10° before and 10° after dead center). The flywheel has a moment of inertia of 300 units of mass-ft.^{2} and the average r.p.m. is 200. What is the variation in speed above and below the average?

Problem 13–25. An engine delivers 40,000 ft.-lbs. of work in 90 seconds. What is the horse-power?

Problem 13–26. If the engine of the preceding problem is turning at 200 r.p.m. what is the torque at the shaft? *Ans.* 21.2 lb.-ft.

Problem 13–27. An auto engine delivers 300,000 ft.-lbs. of work per minute, the engine shaft revolving at 1000 r.p.m. The engine shaft revolves 3.5 times as fast as the rear wheels to which it is geared. If 35,000 ft.-lbs. of work per minute is lost in friction what is the efficiency?

Problem 13–28. What is the torque at the rear axle in the preceding problem? *Ans.* 147.4 lb.-ft.

PROBLEM 13–29. What is the tractive force in the preceding problem if the diameter of the rear wheels is 30 ins? *Ans.* 117.2 lbs.

PROBLEM 13–30. A double-acting steam engine has a cylinder 12 ins. in diameter, a stroke of 18 ins. and a piston rod diameter of 3 ins. The r.p.m. is 200. If the mean effective pressure is 52.5 lbs. per sq. in. on the head end and 58.2 lbs. per sq. in. on the crank end, how much work is done per revolution on each end?

PROBLEM 13–31. In the preceding problem how much work is done per minute?

PROBLEM 13–32. What is the Ihp. on the head end? On the crank end? What is the total Ihp. delivered?

PROBLEM 13–33. Check the results of the preceding problem by the approximate formula.

PROBLEM 13–34. If a double-acting engine has a piston speed of 1000 ft. per min., a mean effective pressure in each end of 65 lbs. per sq. in., a crank end area of 22.7 sq. ins. and a head end area of 25 sq. ins., what is its Ihp.?

PROBLEM 13–35. A double-acting engine has a mean effective pressure of 63 lbs. per sq. in., a piston diameter of 8 ins. and an r.p.m. of 225. Neglecting the diameter of the piston rod what is the Ihp.? *Ans.* 39.4 hp.

PROBLEM 13–36. A four-cylinder four-cycle auto engine having a bore of $3\frac{3}{4}$ ins., a stroke of 4 ins. and r. p. m. of 1500 has a mean effective pressure of 74.5 lbs. per sq. in. What is the Ihp.? *Ans.* 24.8 hp.

PROBLEM 13–37. If the mechanical efficiency in the preceding problem is 90 per cent what is the torque at the shaft? *Ans.* 78 lb.-ft.

PROBLEM 13–38. If the transmission efficiency is 80 per cent and the gear ratio is $3\frac{7}{11}$ to 1 what is the torque at the axle in the preceding problem? *Ans.* 227 lb.-ft.

PROBLEM 13–39. A six-cylinder four-cycle Standard marine engine, 10-in. bore, 11-in. stroke, developed 220 Bhp. at 460 r.p.m. Assuming a mechanical efficiency of 90 per cent what was the mean effective pressure? *Ans.* 80.9 lbs./sq. ins.

PROBLEM 13–40. A Prony brake has an unbalanced weight of 60 lbs. and an arm of 4 ft. The scale reading is 450 lbs. when the engine speed is 104 r.p.m. What is the Bhp.? *Ans.* 30.8.

PROBLEM 13–41. The rear wheel of an automobile is removed and a Prony brake having a beam 6 ft. long, a pulley 30 ins. in diameter, and an unbalanced weight of 10 lbs. is put in its place. What is the scale reading of this brake if the engine is averaging 74 lbs. per sq. in. mean effective pressure at an r.p.m. of 1500. Mechanical efficiency 85 per cent, ratio shaft turns to axle turns $3\frac{7}{11}$ to 1, transmission efficiency 80 per cent. Area of pistons of engine 11 sq. ins., stroke 4 ins. The engine has four cylinders and is four-cycle. *Ans.* 45.7 lbs.

Degrees	Sin	Cos	Tan	
0° 00′	.0000	1.0000	.0000	90° 00′
0° 10′	.0029	1.0000	.0029	89° 50′
0° 20′	.0058	1.0000	.0058	89° 40′
0° 30′	.0087	1.0000	.0087	89° 30′
0° 40′	.0116	.9999	.0116	89° 20′
0° 50′	.0145	.9999	.0145	89° 10′
1° 00′	.0175	.9998	.0175	89° 00′
1° 10′	.0204	.9998	.0204	88° 50′
1° 20′	.0233	.9997	.0233	88° 40′
1° 30′	.0262	.9997	.0262	88° 30′
1° 40′	.0291	.9996	.0291	88° 20′
1° 50′	.0320	.9995	.0320	88° 10′
2° 00′	.0349	.9994	.0349	88° 00′
2° 10′	.0378	.9993	.0378	87° 50′
2° 20′	.0407	.9992	.0407	87° 40′
2° 30′	.0436	.9990	.0437	87° 30′
2° 40′	.0465	.9989	.0466	87° 20′
2° 50′	.0494	.9988	.0495	87° 10′
3° 00′	.0523	.9986	.0524	87° 00′
3° 10′	.0552	.9985	.0553	86° 50′
3° 20′	.0581	.9983	.0582	86° 40′
3° 30′	.0610	.9981	.0612	86° 30′
3° 40′	.0640	.9980	.0641	86° 20′
3° 50′	.0669	.9978	.0670	86° 10′
4° 00′	.0698	.9976	.0699	86° 00′
4° 10′	.0727	.9974	.0729	85° 50′
4° 20′	.0756	.9971	.0758	85° 40′
4° 30′	.0785	.9969	.0787	85° 30′
4° 40′	.0814	.9967	.0816	85° 20′
4° 50′	.0843	.9964	.0846	85° 10′
5° 00′	.0872	.9962	.0875	85° 00′
5° 10′	.0901	.9959	.0904	84° 50′
5° 20′	.0929	.9957	.0934	84° 40′
5° 30′	.0958	.9954	.0963	84° 30′
5° 40′	.0987	.9951	.0992	84° 20′
5° 50′	.1016	.9948	.1022	84° 10′
6° 00′	.1045	.9945	.1051	84° 00′
6° 10′	.1074	.9942	.1080	83° 50′
6° 20′	.1103	.9939	.1110	83° 40′
6° 30′	.1132	.9936	.1139	83° 30′
6° 40′	.1161	.9932	.1169	83° 20′
6° 50′	.1190	.9929	.1198	83° 10′
	Cos	Sin	Cot	Degrees

Degrees	Sin	Cos	Tan	
7° 00′	.1219	.9925	.1228	83° 00′
7° 10′	.1248	.9922	.1257	82° 50′
7° 20′	.1276	.9918	.1287	82° 40′
7° 30′	.1305	.9914	.1317	82° 30′
7° 40′	.1334	.9911	.1346	82° 20′
7° 50′	.1363	.9907	.1376	82° 10′
8° 00′	.1392	.9903	.1405	82° 00′
8° 10′	.1421	.9899	.1435	81° 50′
8° 20′	.1449	.9894	.1465	81° 40′
8° 30′	.1478	.9890	.1495	81° 30′
8° 40′	.1507	.9886	.1524	81° 20′
8° 50′	.1536	.9881	.1554	81° 10′
9° 00′	.1564	.9877	.1584	81° 00′
9° 10′	.1593	.9872	.1614	80° 50′
9° 20′	.1622	.9868	.1644	80° 40′
9° 30′	.1650	.9863	.1673	80° 30′
9° 40′	.1679	.9858	.1703	80° 20′
9° 50′	.1708	.9853	.1733	80° 10′
10° 00′	.1736	.9848	.1763	80° 00′
10° 20′	.1794	.9838	.1823	79° 40′
10° 40′	.1851	.9827	.1883	79° 20′
11° 00′	.1908	.9816	.1944	79° 00′
11° 20′	.1965	.9805	.2004	78° 40′
11° 40′	.2022	.9793	.2065	78° 20′
12° 00′	.2079	.9781	.2126	78° 00′
12° 20′	.2136	.9769	.2186	77° 40′
12° 40′	.2193	.9757	.2247	77° 20′
13° 00′	.2250	.9744	.2309	77° 00′
13° 20′	.2306	.9730	.2370	76° 40′
13° 40′	.2363	.9717	.2432	76° 20′
14° 00′	.2419	.9703	.2493	76° 00′
14° 20′	.2476	.9689	.2555	75° 40′
14° 40′	.2532	.9674	.2617	75° 20′
15° 00′	.2588	.9659	.2679	75° 00′
15° 30′	.2672	.9636	.2773	74° 30′
16° 00′	.2756	.9613	.2867	74° 00′
16° 30′	.2840	.9588	.2962	73° 30′
17° 00′	.2924	.9563	.3057	73° 00′
17° 30′	.3007	.9537	.3153	72° 30′
	Cos	Sin	Cot	Degrees

Degrees	Sin	Cos	Tan	
18° 00′	.3090	.9511	.3249	72° 00′
18° 30′	.3173	.9483	.3346	71° 30′
19° 00′	.3256	.9455	.3443	71° 00′
19° 30′	.3338	.9426	.3541	70° 30′
20° 00′	.3420	.9397	.3640	70° 00′
20° 30′	.3502	.9367	.3739	69° 30′
21° 00′	.3584	.9336	.3839	69° 00′
21° 30′	.3665	.9304	.3939	68° 30′
22° 00′	.3746	.9272	.4040	68° 00′
22° 30′	.3827	.9239	.4142	67° 30′
23° 00′	.3907	.9205	.4245	67° 00′
23° 30′	.3987	.9171	.4348	66° 30′
24° 00′	.4067	.9135	.4452	66° 00′
24° 30′	.4147	.9100	.4557	65° 30′
25° 00′	.4226	.9063	.4663	65° 00′
26° 00′	.4384	.8988	.4877	64° 00′
27° 00′	.4540	.8910	.5095	63° 00′
28° 00′	.4695	.8829	.5317	62° 00′
29° 00′	.4848	.8746	.5543	61° 00′
30° 00′	.5000	.8660	.5774	60° 00′
31° 00′	.5150	.8572	.6009	59° 00′
32° 00′	.5299	.8480	.6249	58° 00′
33° 00′	.5446	.8387	.6494	57° 00′
34° 00′	.5592	.8290	.6745	56° 00′
35° 00′	.5736	.8192	.7002	55° 00′
36° 00′	.5878	.8090	.7265	54° 00′
37° 00′	.6018	.7986	.7536	53° 00′
38° 00′	.6157	.7880	.7813	52° 00′
39° 00′	.6293	.7771	.8098	51° 00′
40° 00′	.6428	.7660	.8391	50° 00′
41° 00′	.6561	.7547	.8693	49° 00′
42° 00′	.6691	.7431	.9004	48° 00′
43° 00′	.6820	.7314	.9325	47° 00′
44° 00′	.6947	.7193	.9657	46° 00′
45° 00′	.7071	.7071	1.0000	45° 00′
	Cos	Sin	Cot	Degrees

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